MATHS NOTES (1) BY SANDEEP BOHAY

- ✓ VEDIC MATH TRICKS
- ✓ NUMBER SYSTEM
- ✓ SURDS & INDICES
- ✓ NUMBER SERIES
- ✓ EQUATIONS
- ✓ RATIO & ALLIGATION
- ✓ AVERAGE
- ✓ PROFIT & LOSS
- ✓ TIME & WORK
- ✓ PERMUTATION,COMBINATION,PROBABILITY
- ✓ MISCELLANEOUS

VEDIC MATH TRICKS

1] TO FIND SQUARE of ANY NUMBER :-

Method 1 :- $a^2 | 2ab | b^2$ → three parts.

$(37)^2$ → $a^2 | 2ab | b^2$

a b

9 | 42 | 49

13 46

1369

$(43)^2$ → $4^2 | 2\times4\times3 | 3^2$

16 | 24 | 9

18

1849

$(76)^2$ → 49 | 84 | 36

5776

$(89)^2$ → 64 | 144 | 81

79 15 2

7921

NOTE:- Take rightmost digit while combining the parts & start from right side.

$(137)^2$ → $13^2 | 2\times13\times7 | 7^2$

a b

169 | 182 | 49

187 186

18769

$(235)^2$ → 529 | 230 | 25

a b

55225

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Method 2 :- To find square of any no. b/w 1 to 100.

$50^2 = 2500$

↓ 1-50

$(46)^2$ → 50 − 46 = 4

25 − 4 | 4^2

21 | 16

2116 Ans.

1) Check the difference of the no. from 50.
2) Subtract diff. from 25 & write in Ist Part
3) Write square of diff in 2nd Part
4) Combine both parts

51-100

$(57)^2$ → 57 − 50 = 7

25+7 | 7^2 → 32 | 49

3249

1) Check diff. of 50 from the no.
2) Add the diff in 25 & write in Ist Part.
3) Write square of diff. in 2nd Part.
4) Combine both Parts.

NOTE:- There should be two digits in 2nd Part.

→ square of no.s below 50 will always be below 2500 so we will subtract from 25 & square of no.s above will be above 2500 so we will add in 25

$(43)^2 \rightarrow 25-7 \mid 7^2$ → 1849

$(47)^2 \rightarrow 25-3 \mid 3^2 \rightarrow 22 \mid 09 \rightarrow$ 2209

$(38)^2 \rightarrow 25-12 \mid 12^2$ → 13 | 144 → 1444

$(34)^2 \rightarrow 25-16 \mid 16^2$ → 9 | 256 → 1156

$(31)^2 \rightarrow 6 \mid 361$ → 961

$(54)^2 \rightarrow 25+4 \mid 4^2$ → 2916

$(59)^2 \rightarrow 25+9 \mid 9^2$ → 3481

$(64)^2 \rightarrow 25+14 \mid 14^2$ → 39 | 96 → 4096

$(52)^2 \rightarrow 27 \mid 04$ → 2704

$(67)^2 \rightarrow 42 \mid 89$ → 4489

$(72)^2 \rightarrow 25+22 \mid 22^2$ → 47 | 484 → 5184

METHOD 3:- To find square of any no. near base (10, 100 etc)

Base → 100

$(94)^2 \rightarrow$ 100 − 94 = 6

94 − 6 = 88 | 6^2 = 36

→ 8836

$(108)^2 \rightarrow$ 108 − 100 = 8

108 + 8 = 116 | 8^2 = 64

→ 11664

$88 \rightarrow -12$

76 | 144

→ 7744

$97 \rightarrow -3$

94 | 09

→ 9409

$113 \rightarrow +13$

126 | 169

→ 12769

$(117)^2 \rightarrow +17$

134 | 289

→ 13689

METHOD 4:- Square of any no. ending with 5.

$(35)^2 \rightarrow 3\times4\,|\,25$ → 1225

$(55)^2 \rightarrow 5\times6\,|\,25$ → 3025

$(75)^2 \rightarrow 7\times8\,|\,25$ → 5625

$(65)^2$ (×7) → 4225

$(95)^2$ (×10) → 9025

$(105)^2$ (×11) → 11025

$(125)^2$ (×13) → 15625

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2 → TO FIND CUBE

$(a+b)^3 \rightarrow a^3+b^3+3a^2b+3ab^2$

$(ab)^3 \rightarrow a^3\,|\,3a^2b\,|\,3ab^2\,|\,b^3$ [Remember in this form]

$(13)^3 \rightarrow 1^3\,|\,3\times1^2\times3\,|\,3\times1\times3^2\,|\,3^3$ (a, b)

1 | 9 | 27 | 27

2 11 39

2197

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$(24)^3 \rightarrow 2^3\,|\,3\times2^2\times4\,|\,3\times2\times4^2\,|\,4^3$ (a, b)

8 | 48 | 96 | 64

13 58 102

13 8 24

$(36)^3 \rightarrow 27\,|\,162\,|\,324\,|\,216$

+19 +34 +2

46 96 345

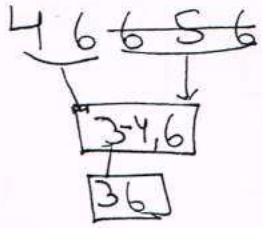
46656

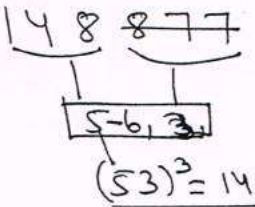
$(53)^3 \rightarrow 125\,|\,225\,|\,135\,|\,27$

148877

3 → To find Cube-root

$1^3 \rightarrow 1$
$2^3 \rightarrow 8$
$3^3 \rightarrow 27$
$4^3 \rightarrow 64$
$5^3 \rightarrow 125$
$6^3 \rightarrow 216$
$7^3 \rightarrow 343$
$8^3 \rightarrow 512$
$9^3 \rightarrow 729$





1) Check the last digit of the given no. & cut last (3) digits.
2) 6 is the last digit of 6^3 & 7 is the last digit of 3^3.
3) Check the rem. digits in table.
eg → 46 comes between 3^3 & 4^3
148 comes between 5^3 & 6^3
Take the smaller one.

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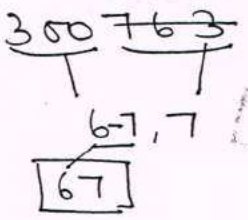
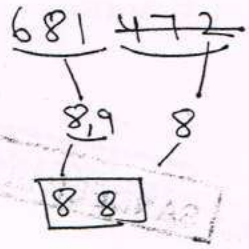
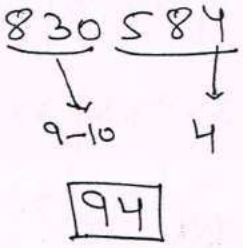
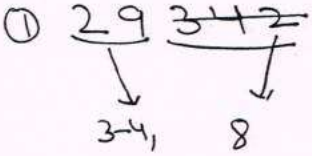


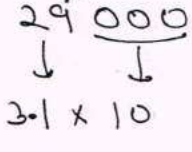


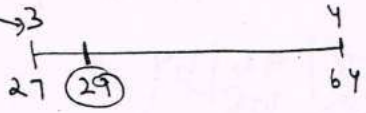


* Approximate Cube-root.

① 29342 → 29000

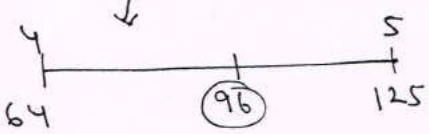






[Cube-root of 29342 & 29000 will be very near]

② 96768



96768
↓
4.5 × 10
45

③ 389629

389629
↓
7.3 or 7.2
↓ ↓
73 or 72

4 :- SQUARE ROOT

$1^2 \rightarrow 1$
$2^2 \rightarrow 4$
$3^2 \rightarrow 9$
$4^2 \rightarrow 16$
$5^2 \rightarrow 25$
$6^2 \rightarrow 36$
$7^2 \rightarrow 49$
$8^2 \rightarrow 64$
$9^2 \rightarrow 81$

In this table, two no.s have the same last digit except 5. eg → $1^2 = 1$, $9^2 = 81$ (both have unit digit 1)

1|9, 2|8, 3|7, 4|6.

Example → 5 7 7 6 (cut last ② digits)

$7^2 - 8^2$ 4|6

①4|⑨6

Confirm no. = 7

$7 \times 8 = 56$

$57 > 56$

So bigger no. is Ans = 76

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8 6 4 9

$9^2 - 10^2$ 3|7

93|97

$9 \times 10 = 90$

$86 < 90$

So 93 Ans.

1 4 4 4

3-4 2|8

32|38

$3 \times 4 = 12$

$14 > 12$

So 38 Ans.

7 9 2 1

8-9 1|9

81|89

$8 \times 9 = 72$

$79 > 72$

So 89 Ans.

* Approximate Square root (Similar to approx cube-root)

3178 → 5.5 or 5.6

5 — 31 — 6

25 — 36

55 or 56

7837 → 8.8 → 88

8 — 78 — 9

64 — 81

27256 → 16.5 or 16.4

165 or 164

16 — 272 — 17

256 — 289

5 → MULTIPLICATION

* Pattern Method (Very useful & easy method of multiplication)

* 2X2 Pattern [Total Parts = n-1 = 3]

Ex:- 3 6 × 7 4

①	②	③
3 6 ↕ 4	3×6 + 7×4	3 6 ↕ 7 4
21	12+42 = 54	24

2664

(6 4 × 3 8)

18	48+12 = 60	32

2432

8 7 × 6 9

48	72+42 = 114	63
	120	

6003

* 3 X 2 Pattern [n-1 = 5-1 = 4 Parts]

2 3 6 × 4 8

2 * * ↕ 4 *	23* × 48	*36 × 48	**6 × 8
8	16+12 = 28	24+24 = 48	48
	33	52	

11328

6 4 7 × 7 3

42	18+28 = 46	12+49 = 61	21

47231

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* 3X3 Pattern [S Parts = n-1]

3 5 6 × 4 6 7

3 * * ↕ 4 * *	35* × 46*	356 × 467	*56 × *67	** 6 ↕ ** 7
12	18+20 = 38	21+24+30 = 75	35+36 = 71	42

166252

4 6 5 × 3 8 7

12	32+18 = 50	28+15+48 = 91	42+40 = 82	35

179955

* 4X2 Pattern [5 Parts = 6-1]

6452 X74

6	64	45	52	28
42	24+28 52	16+35 51	20+14 34	8

477448

7563 X37

21	49+15 64	35+18 53	42+9 51	21

27983

* Multiplication using base method :- (100 Base)

Base → 100

96 X 94 : -4, -6

90 | 24

96-6=90 or 94-4=90

9024

88 X 93 : -12, -7

81 | 84

8184

86 X 89 : -14, -11

75 | 154

7654

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108 X 109 : +8, +9

117 | 72

11772

112 X 109 : +12, +9

121 | 108

12208

104 X 102 : +4, +2

106 | 08

10608

112 X 93 : +12, -7

105 | 84

-1 | ↓

104 | 16 (compliment)

10416

989 X 990 : -11, -10

979 | 110

979110

* Square of a no. near known no. square (Next & previous)

40 → 1600
+41 / 81 — +81 → 1681

70 → 4900
69 / 139 — −139 → 4761

85 → 7225
+86 / 171 — +171 → 7396

140 → 19600
+141 / 281 — +281 → 19881

26 → 676
+25 / 51 — +51 → 625

125 → 15625
+126 / 251 — +251 → 15876

* Multiplication with 5, 25, 125, 625

5 → $\frac{10}{2}$, 25 → $\frac{100}{4}$, 125 → $\frac{1000}{8}$, 625 → $\frac{10000}{16}$

It means instead of multiplying a no. with 25, divide it by 4 & put two zeros in the end, In case of 125 & 625 divide the no. by 8 & 16 resp. & put three zeros & four zeros in end resp.

Ex:- $64 \times 25 \rightarrow \frac{64}{4} = 1600$; $93 \times 25 \rightarrow \frac{93}{4} = 23.25 \times 100 = \boxed{2325}$

$96 \times 125 \rightarrow \frac{96}{8} = \boxed{12000}$, $122 \times 125 \rightarrow \frac{122}{8} = 15.25 \times 1000 = \boxed{15250}$

$272 \times 625 \rightarrow \frac{272}{16} = \boxed{170000}$, $164 \times 625 \rightarrow \frac{164}{16} = 10.25 \times 10000 = \boxed{102500}$

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* Compliment of a no.:-

2 → 8 (10−2)
4 → 6 (10−4)
7 → 3 (10−7)

18 → 82 (100−18)
32 → 68 (100−32)
46 → 54 (100−46)
77 → 23 (100−77)

132 → 868 (1000−132)
371 → 629 (1000−371)
555 → 445 (1000−555)
843 → 157 (1000−843)

✱ Second Method of square-root :-

1st reference → $(100)^2$ → 10,000

2nd reference → $(50)^2$ → 2500

When Base is $(100)^2$ → 10000 :-

① **Below 10,000** :- check last 2 digits everytime.

→ 9604 :- 04 is square of 2 so subtract 2 from 100 because of sq. root of 9604 will be below 100.

100 − 2 = **98**

→ 8649 → 7^2 → 100 − 7 = **93**

7644 → $12^2 \to 144$ → 100 − 12 = **88**

6889 → $17^2 \to 289$ → 100 − 17 = **83**

② **Above 10,000** :-

11236 → 6^2 → 100 + 6 = **106**

12769 → $13^2 \to 169$ → 100 + 13 = **113**

14161 → $19^2 \to 361$ → 100 + 19 = **119**

③ **Below 2500** (50^2)

1764 → 8^2 → 50 − 8 = **42**

1369 → $13^2 \to 169$ → 50 − 13 = **37**

2209 → 3^2 → 50 − 3 = **47**

④ **Above 2500**

3481 → 9^2 → 50 + 9 = **59**

4356 → $16^2 = 256$ → 50 + 16 = **66**

2704 → 2^2 → 50 + 2 = **52**

✱ Multiplication with 9, 99, 999 ... (Two parts)

(i) 8 × 9

I: 8 − 1 | II: 2 → compliment of 8

72

Compliment of 8 → 2 | 3 → 7 | 82 → 18

14 → 86 | 74 → 26 | 55 → 45 etc.

282 → 718 | 460 → 540 | 864 → 136

(ii) 75 × 99

75 − 1 | 25 → compliment of 75

7425

(iii) 737 × 999

737 − 1 | 263 → Compliment of 737

736263

(IV) $3\,37 \times 99$

$337-(3+1) \mid 63$ → comp. of 37

333 63

(V) 1442×99

$1442-15 \mid 58$ → comp of 42

1427 58

(VI) 8649×999

$8649-9 \mid 351$ → comp. of 649

8640351

(VII) 168×9

$168-17 \mid 2$ → comp. of 8

1512

* Alternate method of multiply with 9, 99, 999 etc.

$75 \times 99 \rightarrow 75(100-1) \rightarrow 7500-75 \rightarrow$ 7425

$337 \times 99 \rightarrow 337(100-1) \rightarrow 33700-337 \rightarrow$ 33363

$8649 \times 999 \rightarrow 8649(1000-1) \rightarrow 8649000-8649 \rightarrow$ 8640351

* Multiplication with 11, 111 :-

68×11

6 | 6+8 | 8

14

748

734×11

7 | 7+3 | 3+4 | 4

10 7

8074

$3\,6\,8 \times 11$

3+4 | 4+2 | 2+8 | 8

7 6 10

37708

467×11

5 | 3 | 7

5137

347×111

3 | 3+4 | 3+4+7 | 7+4 | 7

7 14 11

38517

645×111

7 | 5 | 9 | 5

71595

4673×111

5 | 8 | 7 | 0 | 3

518703

Imp - $777 \times 77 \times 7$ or $666 \times 66 \times 6$ etc.

$7 \times 111 \times 7 \times 11 \times 7 \times 1$

$343 \times 111 \times 11$

343×111

3 | 8 | 0 | 7 | 3

38073×11

38073×11

4 | 1 | 8 | 8 | 0 | 3

NUMBER SYSTEM

Q1 Find the number of zeros in the end?

(a) $1 \times 2 \times 3 \times 4 \times \text{---} \times 700 = ?$

Shortcut:- $\frac{700}{5} = \frac{140}{5} = \frac{28}{5} = \frac{5}{5} = 1 \Rightarrow 140+28+5+1 = \boxed{174}$ zeros

(b) $2 \times 4 \times 6 \times 8 \times \text{---} \times 180 = ?$

$\Rightarrow \frac{180}{2} \Rightarrow \frac{90}{5} = \frac{18}{5} = 3 \rightarrow 18+3 = \boxed{21}$ zeros

(c) $3 \times 6 \times 9 \times 12 \times \text{---} \times 450$

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$\frac{450}{3} \Rightarrow \frac{150}{5} = \frac{30}{5} = \frac{6}{5} = 1 \rightarrow 30+6+1 = \boxed{37}$ Ans.

(d) $\boxed{110}$ or $110!$ (factorial) = ?

$110! \rightarrow 110 \times 109 \times 108 \times \text{---} \times 1$

$\frac{110}{5} \Rightarrow \frac{22}{5} = 4 \rightarrow 22+4 = \boxed{26}$ Ans.

Note:- if all the nos are odd then there will not be any zero in products end.

ex:- $1 \times 3 \times 5 \times 7 \times \text{---} \times 99$

number of zeroes = 0

(e) $\boxed{76} \times \boxed{96} \times \boxed{162} = ?$

$\frac{76}{5} = \frac{15}{5} = 3 \rightarrow \boxed{18}$ $\quad \frac{96}{5} = \frac{19}{5} = 3 \rightarrow 22$

$\frac{162}{5} = \frac{32}{5} = \frac{6}{5} = 1 \rightarrow \boxed{39}$ $\quad$ Total $\Rightarrow 18+22+39 = \boxed{79}$ Ans.

Q2

(a) $\frac{1}{1\times4} + \frac{1}{4\times7} + \frac{1}{7\times10} + \frac{1}{10\times13} + \frac{1}{13\times16} = ?$

(a) ← (b)

$\rightarrow \frac{1}{3}\left[\frac{3}{1\times4} + \frac{3}{4\times7} + \text{---} + \frac{3}{13\times16}\right] = \frac{1}{3}\left[\frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10} + \frac{1}{10} - \frac{1}{13} + \frac{1}{13} - \frac{1}{16}\right]$

$\rightarrow \frac{1}{3}\left[1 - \frac{1}{16}\right] \rightarrow \frac{1}{3} \times \frac{15}{16} = \boxed{\frac{5}{16}}$

Shortcut:- $\frac{\text{No. of terms}}{a \times b}$

$= \frac{5}{1\times16} = \boxed{\frac{5}{16}}$ Ans.

(b) $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \text{---} + \frac{1}{110} = ?$

$\rightarrow \frac{1}{1\times2} + \frac{1}{2\times4} + \frac{1}{4\times5} + \text{---} + \frac{1}{10\times11}$

Shortcut $= \frac{9}{2\times11} = \boxed{\frac{9}{22}}$ Ans.

(c) $\frac{3}{1\times4} + \frac{5}{4\times9} + \frac{7}{9\times16} + \frac{9}{16\times25} + \cdots\cdots\cdots + \frac{19}{81\times100} = ?$

$\rightarrow \frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{16} + \cdots\cdots\cdots\cdots - \frac{1}{81} - \frac{1}{100}$

$\Rightarrow \frac{1}{1} - \frac{1}{100} = \boxed{\frac{99}{100}}$ Ans.

(d) $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\cdots\cdots\left(1+\frac{1}{11}\right)$

$= \frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\frac{6}{5}\times\cdots\cdots\frac{12}{11} \Rightarrow \frac{12}{2} = \boxed{6}$ Ans.

Q3 (a) A no. when divided by 57 leaves the remainder 28. if the same no. is divided by 19, what will be remainder?

$\therefore$ Short cut:- $19\overline{)28(1}$
19
$\boxed{9}$ Ans.

$57\overline{)x(k}$
$\frac{1}{28}$

$x = \frac{57k+28}{19} \rightarrow \frac{3}{\cancel{57}k} + \frac{28}{19}$

$\frac{28}{19} \rightarrow$ Rem $= \boxed{9}$

(b) A no. when divided by 72 leaves the remainder 22. if the thrice of same no. is divided by 12, what will be remainder?

Short cut:- $22\times3 = 66 \rightarrow 12\overline{)66(5}$
60
$\boxed{6}$ Ans.

(c) A no. when divided by 84 leaves remainder 5. if the square of same no. is divided by 14, what will be remainder?

Short cut:- $(5)^2 = 25 \rightarrow 14\overline{)25(1}$
14
$\boxed{11}$ Ans.

Q4 (a) Find the no. of prime factors of 144?

$\rightarrow 144 = 12\times12$
$= 2^2\times3^1\times2^2\times3^1$
$= 2^4\times3^2$

Ans $\rightarrow \boxed{4+2=6}$

$225 \rightarrow 5^2\times3^2$

$\boxed{2+2=4}$ Ans.

→ No. of prime factors of $(6)^3 \times (5)^2 \times (21)^4$?

$(2\times3)^3 \times (5)^2 \times (3\times7)^4$ → All bases are prime nos

~~Ans~~ → $2^3 \times 3^3 \times 5^2 \times 3^4 \times 7^4$ → Ans ⇒ 3+3+2+4+4 = 16

(b) Find the total no. of factors of 72 ?

→ $72 = 2^3 \times 3^2 \rightarrow x^a \times y^b$

Shortcut:- $(a+1)\times(b+1) \Rightarrow (3+1)\times(2+1) =$ 12

→ $360 = 2^3 \times 3^2 \times 5^1 \Rightarrow (3+1)\times(2+1)\times(1+1) =$ 24

→ Total no. of factors of 360 except 1 & itself ⇒ 24−2 = 22

Q.5 To find the UNIT DIGIT ?

(a) Find unit digit in $32①\times 45⑥ \times 25⑧ \times 75③$?

→ $1\times6\times8\times3$ → 4 Ans.

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(b) Find unit digit in $(32\underline{8})^{58}$?

x

Method :- 1) Check last digit i.e. x.

2) if it is 0, 1, 5, 6, then it is the Ans itself

3) if any other no., then divide power by 4 & check remainder.

4) if rem = 1 then x^1
if rem = 2 then x^2
if rem = 3 then x^3 & then solve
if rem = 0 then x^4

$(328)^{58}$ → $\frac{58}{4}$ → Rem = 2 ⇒ 8^2 → 6④ → 4 Ans.

⇒ $(256)^{131} \times (327)^{30} \times (152)^{295} \times (73)^{76} \times (131)^{228}$?

$\frac{30}{4}$ → R = 2 ; $\frac{295}{4}$ → R = 3 ; $\frac{76}{4}$ ⇒ R = 0

6 ; $7^2 = 49$; $2^3 = 8$; $3^4 = 81$; 1

5④ ; 3② ; 2 ; 2

Ans = 2

⑤⑥ Recurring Decimals:-

Ex → $3\overline{)20}(6.6666\ldots$ $\quad \frac{20}{3} = 6.666\ldots \Rightarrow 6.\overline{6}$ (Bar)

$\frac{18}{20}$
$\frac{18}{2}$

Changing recurring decimals into fractions:-

(a) $0.444\ldots \rightarrow 0.\overline{4} \rightarrow \boxed{\frac{4}{9}}$

(b) $0.474747\ldots \rightarrow 0.\overline{47} \rightarrow \boxed{\frac{47}{99}}$

(c) $0.512512\ldots \rightarrow 0.\overline{512} \rightarrow \boxed{\frac{512}{999}}$

(d) $0.4333\ldots \rightarrow 0.4\overline{3} \rightarrow \frac{43-4}{90} \rightarrow \boxed{\frac{39}{90}}$

(e) $0.32161616\ldots \rightarrow 0.32\overline{16} \rightarrow \frac{3216-32}{9900} \rightarrow \boxed{\frac{3184}{9900}}$

(f) $6.666\ldots \rightarrow 6+0.\overline{6} \rightarrow 6+\frac{6}{9} \rightarrow 6+\frac{2}{3} \rightarrow \boxed{\frac{20}{3}}$

(g) $0.2\overline{89} \rightarrow \frac{289-2}{990} \rightarrow \boxed{\frac{287}{990}}$

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(h) $7.2\overline{89} \rightarrow 7+0.289 \rightarrow \boxed{7+\frac{287}{990}}$

---

* if Bar comes before point then that digit will be negative.

$\overline{6}.43 = -6+0.43 = \boxed{-5.57}$ Ans.

$\overline{37}.28 = -37+0.28 = \boxed{-36.72}$ Ans.

---

* Solve:- $7.\overline{53} + 6.\overline{04} - 3.\overline{24}$?

→ $(7+6-3) + \frac{53}{99} + \frac{4}{99} - \frac{24}{99} = 10 + \frac{331}{99_{33}} = \boxed{\frac{31}{3}}$ Ans.

* Solve:- $0.\overline{32} + 0.\overline{4} + 0.00\overline{2}$

→ 0.32323232
0.44444444
0.00222222
0.76969698

$\boxed{0.76\overline{98}}$

Solve:- $0.0\overline{3} + 0.\overline{2} + 0.00\overline{4}$

→ 0.03333
0.22222
0.00444
0.25999

$\boxed{0.25\overline{9}}$

Q⑦ $13\frac{2}{3} + 15\frac{4}{5} - 7\frac{13}{15} + 12\frac{11}{20}$?

$\rightarrow (13+15-7+12) + \left(\frac{2}{3}+\frac{4}{5}-\frac{13}{15}+\frac{11}{20}\right)$

$\rightarrow 33 + \left(\frac{40+48-52+33}{60}\right) \Rightarrow 33 + \frac{69}{60} \Rightarrow 33 + 1\frac{9}{60}$

$\Rightarrow 34 + \frac{9}{60_{20}}^{3} \Rightarrow \boxed{34\frac{3}{20}}$ Ans.

$189\frac{42}{47} + 289\frac{43}{47} + 389\frac{44}{47} - 219\frac{37}{47} - 125\frac{13}{47} = ?$

$\Rightarrow (189+289+389-219-125) + \left(\frac{42}{47}+\frac{43}{47}+\frac{44}{47}-\frac{37}{47}-\frac{13}{47}\right)$

$\rightarrow (523) + \left(\frac{79}{47}\right) \Rightarrow 523 + 1\frac{32}{47} \Rightarrow \boxed{524\frac{32}{47}}$ Ans.

Q⑧ Comparison of fractions :-

(a) Which is bigger among $\frac{7}{8}, \frac{9}{10}$?

$\rightarrow \frac{7}{8} \times \frac{9}{10}$ so $\boxed{\frac{7}{8} < \frac{9}{10}}$

$70 < 72$

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(b) $\frac{13}{15} \times \frac{11}{12}$ so $\boxed{\frac{11}{12} > \frac{13}{15}}$

$156 < 165$

(c) Arrange in Ascending order :- $\frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{5}{12}, \frac{8}{15}$

The Best method is to put a zero in numerator & then divide → $\frac{30}{7}, \frac{40}{9}, \frac{50}{11}, \frac{50}{12}, \frac{80}{15}$

[upto 1 place of decimal only] $4.2, 4.4, 4.5, 4.1, 5.3$

Ascending order → $\frac{5}{12} < \frac{3}{7} < \frac{4}{9} < \frac{5}{11} < \frac{8}{15}$ (Highest)

(d) Descending order :- of $\frac{4}{9}, \frac{5}{14}, \frac{1}{2}, \frac{3}{4}, \frac{2}{3}$

$\rightarrow \frac{40}{9}, \frac{50}{14}, \frac{10}{2}, \frac{30}{4}, \frac{20}{3}$

$4.4, 3.5, 5, 7.25, 6.6$

$\Rightarrow \frac{3}{4} > \frac{2}{3} > \frac{1}{2} > \frac{4}{9} > \frac{5}{14}$ (lowest)

Q(9) if $2^x = 3^y = 6^{-z}$ then $\boxed{\frac{1}{x}+\frac{1}{y}=\frac{-1}{z}}$ or $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

Proof:- $2^x = 3^y = 6^{-z} = k$

$2^x = k \rightarrow 2 = k^{1/x}$

$3^y = k \rightarrow 3 = k^{1/y}$

$6^{-z} = k \rightarrow 6 = k^{-1/z}$

$2 \times 3 = 6$

$\Rightarrow$ so $k^{1/x} \times k^{1/y} = k^{-1/z}$

$k^{1/x+1/y} = k^{-1/z}$

so $\boxed{\frac{1}{x}+\frac{1}{y}=-\frac{1}{z}}$ Ans.

Q(10) Finding the Remainder ? **SANDEEP BOHAY™**

(a) Any number (x) having even power when divided by its previous no. $(x-1)$ or next no. $(x+1)$, then Remainder = 1.

$\Rightarrow \frac{(17)^{200}}{18} = \frac{(18-1)^{200}}{18} \rightarrow (-1)^{200} = \boxed{1}$ Ans.

$\Rightarrow \frac{(15)^{84}}{14} = \frac{(14+1)^{84}}{14} \rightarrow (1)^{84} = \boxed{1}$ Ans.

$\Rightarrow \frac{(16^{64}+1)}{17} = \frac{(17-1)^{64}+1}{17} \rightarrow 1+1 = \boxed{2}$ Ans.

$\Rightarrow \frac{(17^{90}+18)}{18} = \frac{(18-1)^{90}+18}{18} = \frac{1+18}{18} = \frac{19}{18} \Rightarrow \boxed{R=1}$ Ans

[Rem. can't be greater than divisor]

[$\frac{(x)^{\text{even}}}{(x-1) \text{ or } (x+1)}$ then $R=1$]

(b) $\frac{(x)^{\text{odd}}}{x-1} \rightarrow$ Rem = 1 & $\frac{(x)^{\text{odd}}}{x+1} \rightarrow$ Rem = x

$\Rightarrow \frac{12^{61}}{13} = \frac{(13-1)^{61}}{13} = (-1)^{61} = -1$ so $13-1 = \boxed{12}$ Ans.

$\Rightarrow \frac{14^{31}+1}{15} = \frac{(15-1)^{31}+1}{15} = -1+1 = \boxed{0}$ Ans

$\Rightarrow \frac{17^{81}+1}{16} = \frac{(16+1)^{81}+1}{16} = 1+1 = \boxed{2}$ Ans.

$\Rightarrow \frac{67^{67}+67}{68} = \frac{(68-1)^{67}+67}{68} = \frac{-1+67}{68} = \frac{66}{68} = \boxed{66}$ Ans.

$\Rightarrow \frac{(25)^{25}}{24} \rightarrow \frac{(24+1)^{25}}{24} \rightarrow (1)^{25} = \boxed{1}$ Ans.

Q(11)

(1) Sum of Ist 'n' natural nos = $\frac{n(n+1)}{2}$

(2) Sum of Ist 'n' odd numbers = n^2 → [where → $n = \frac{\text{last term}+1}{2}$]

(3) Sum of Ist 'n' even numbers = $n(n+1)$ → [$n = \frac{\text{last term}}{2}$]

(4) Sum of squares of Ist 'n' natural nos = $\frac{n(n+1)(2n+1)}{6}$

(5) Sum of cubes of Ist n natural nos = $\left[\frac{n(n+1)}{2}\right]^2$

(6) Sum of squares of Ist n even nos = $\frac{2n(n+1)(2n+1)}{3}$

Q:- $1+3+5+7+\dots+49 = ?$

$n = \frac{49+1}{2} = 25$

Ans = $25^2 = \boxed{625}$

Q:- $51+52+53+\dots+99 = ?$

→ $(1-99) - (1-49)$

$n = \frac{99+1}{2} = 50$, $n = \frac{1+49}{2} = 25$

Ans $50^2 - 25^2 \Rightarrow \boxed{1875}$

Q:→ $1-2+3-4+\dots+97-98 = ?$

→ $(1+3+5+\dots+97) - (2+4+6+\dots+98)$

$n = \frac{97+1}{2} = 49$, $n = \frac{98}{2} = 49$

Ans → $49^2 - 49\times50 = \boxed{-49}$

Q:- $52+54+56+\dots+98 = ?$

→ $(2+4+6+\dots+98) - (2+4+6+\dots+50)$

$n = \frac{98}{2} = 49$, $n = \frac{50}{2} = 25$

Ans $49\times50 - 25\times26 = \boxed{1800}$

Q:- $2^2+4^2+6^2+\dots+20^2 = ?$

→ $n = \frac{20}{2} = 10$

Ans:- $\frac{2\times10(10+1)(2\times10+1)}{3}$

$= \frac{20\times11\times21}{3}$

$= \boxed{1540}$

Q:- $1+2+3+\dots+49+50+49+48+47+\dots+2+1 = ?$

→ Shortcut:- $50^2 = \boxed{2500}$ Ans.

Q:- $1+2+3+\dots+26+27+26+25+\dots+2+1 = ?$

→ Ans:- $27^2 = \boxed{729}$ Ans.

Q(12) Consecutive Numbers:-

(1) Consecutive natural nos → $x, x+1, x+2, x+3, ----$

(2) Consecutive odd/even nos → $x, x+2, x+4, x+6, ----$

→ $x, x+2, \underbrace{x+4}_{\text{Average}}, x+6, x+8$ | $x, x+2, \underbrace{x+4, x+6}_{x+5 = \text{Average}}, x+8, x+10$

$$\text{Average} = \frac{\text{Total sum}}{\text{Total numbers}}$$

Q:- Average of 7 consecutive even numbers is 22. Find highest no. & lowest no.?

→ $x, x+2, x+4, \underbrace{x+6}_{\text{Avg.}=22}, x+8, x+10, x+12$

$x+6=22 \rightarrow x=16$

Highest → 16+12 = 28

Lowest ⇒ 16

Shortcut:-

No.
7
−1
6

Avg.		
22	22	
+6	−6	
28 → Highest	16 → Lowest	

Q:- Average of 6 consecutive odd numbers is 30 → Highest?

→

No.	Avg.		
6	30	30	
−1	+5	−5	
5	35 → Highest	25 → Lowest	

Q:- Sum of 6 consecutive odd numbers is 396 → Smallest?

→ Avg = $\frac{396}{6} = 66$ →

No.	Avg
6	66 − 5 = 61 Ans.
−1	
5	

Q:- Avg. of 9 consecutive natural nos is 18. Find Highest of these numbers? →

No.	Avg.		
9	18	18	
−1	+4	−4	
8	22 → Highest	14 → Lowest.	

for natural nos: $\frac{8}{2} = 4$

Q13 AP series

→ $a, a+d, a+2d, a+3d+\cdots$

↓ 1st term, d = common difference.

nth term, $\boxed{T_n = a+(n-1)d}$

Sum of n terms, $S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[a+l]$ ↳ last term.

Q:- Find 27th term of 8, 15, 22, ---- ?

→ $a=8, d=7, n=27 \rightarrow T_{27} = 8+26\times7 = \boxed{190}$

Q:- How many terms between 100 & 400 are divisible by 3?

→ 102, 105, 108, ---399

$a=102, d=3, l=399 \rightarrow 399 = 102+(n-1)\times3$

$\boxed{n=100}$

Shortcut:- $\frac{400}{3} - \frac{100}{3} \rightarrow 133-33 = \boxed{100}$

Q:- How many numbers between 100 & 200 are div. by 8?

→ $\frac{200}{8} - \frac{100}{8} \Rightarrow (25-1)-12 = \boxed{12}$

[1 is subtracted from 25 because 200 is div. by 8 but we have to exclude 200 because of 'between']

Q:- How many numbers between 500 & 1000 are divisible by 2, 3 & 4?

→ LCM of 2, 3 & 4 = 12 $\Rightarrow \frac{1000}{12} - \frac{500}{12} \rightarrow 83-41 = \boxed{42}$

$12\overline{)1000}(83$; $12\overline{)500}(41$

96 ; 48

40 ; 20

36 ; 12

4 ; 8

Q:- How many terms from 200 to 600 are divisible by 4, 5 & 6?

→ LCM of 4, 5, 6 → 60 $\Rightarrow \frac{600}{60} - \frac{200}{60} \Rightarrow 10-3 = \boxed{7}$

[Here 'from' is used in place of between so 600 is also included]

Q14 Successive Divisibility

Q:- What will be the remainders when 343 is successively divided by 5, 6 & 7?

→ 5 | 343 | 68 — 6 | 68 | 11 — 7 | 11 | 1

30, 43, 40, 3 — 6, 8, 6, 2 — 7, 4

Ans→ 3, 2, 4

Or

		Rem.↓
5	343	
6	68	3
7	11	2
	1	1

Ans→ 3, 2, 4

Q:- Find that smallest no. which when divided by 12 & 15 successively gives rem. 8 & 10 respectively?

→

		Rem.↓
12	X	
15	Y	8
	1	10

$Y = 15 \times 1 + 10 = 25$

$X = (12 \times 25) + 8 = 308$

Q:- When a no. is divided by 4, 6 & 8 then their successive Remainders are 2, 3 & 4. If the order of Divisors is reversed (i.e. 8, 6, 4) then what will be the remainders?

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		Rem.↓
4	X	
6	Y	2
8	Z	3
	1	4

$Z = 8 \times 1 + 4 = 12$

$Y = 6 \times 12 + 3 = 75$

$X = 4 \times 75 + 2 = 302$

→

		Rem.↓
8	302	
6	37	6
4	6	1
	1	2

Ans→ 6, 1, 2

Note 1:- The difference of squares of two consecutive natural nos is equal to the sum of those numbers?

Ex:- $13^2 = 169$, $12^2 = 144 \Rightarrow 169 - 144 = 25$ i.e. $13 + 12$

Note 2:- The difference of squares of two consecutive even numbers is always divisible by 4 & in case of odd nos. it is always divisible by 8.

Ex:- $11^2 - 9^2 = \frac{40}{8} = 5$ | $10^2 - 8^2 = \frac{36}{4} = 9$

Q15 Surds - Indices

(1) $\sqrt{12+\sqrt{12+\sqrt{12+\cdots\infty}}}$

12 → 3, (4) ← Ans.

if sign '+' then bigger factor Ans.

$\sqrt{20-\sqrt{20-\sqrt{20-\cdots\infty}}}$

20 → (4), 5

if sign '−' then smaller factor Ans.

$x=\sqrt{12+x} \rightarrow x^2-x-12=0$

$(x+3)(x-4)=0$

$x=-3, (+4)$ ←

(2) $\sqrt{12\sqrt{12\sqrt{12\cdots\infty}}}$

Ans = 12

$n=\sqrt{12n}$

$n^2=12n \rightarrow n=12$

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(3) $\sqrt{12\sqrt{12\sqrt{12\sqrt{12}}}}$

Ans. $(12)^{\frac{2^n-1}{2^n}}$ $n=4$

$\rightarrow 12^{\frac{15}{16}}$

This formula is fixed irrespective of number inside.

(4) Find maximum & minimum from $\sqrt[3]{4}, \sqrt[4]{6}, \sqrt[6]{15}, \sqrt[12]{245}$?

→ LCM of surds i.e. 3, 4, 6 & 12 = 12

$[(4)^{\frac{1}{3}}]^{12} \rightarrow 4^4 \rightarrow 256 \rightarrow$ max.

$[(6)^{\frac{1}{4}}]^{12} \rightarrow 6^3 \rightarrow 216 \rightarrow$ min.

$[(15)^{\frac{1}{6}}]^{12} \rightarrow 15^2 \rightarrow 225$

$[(245)^{\frac{1}{12}}]^{12} \rightarrow 245 \rightarrow 245$

max. = $\sqrt[3]{4}$

min. = $\sqrt[4]{6}$

(5) $\frac{1}{\sqrt{100}+\sqrt{99}} + \frac{1}{\sqrt{99}+\sqrt{98}} + \frac{1}{\sqrt{98}+\sqrt{97}} + \cdots + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{1}} = ?$

→ Shortcut :- 1st − Last = $\sqrt{100}-\sqrt{1} = 10-1 = 9$

OR by rationalization $\frac{1}{\sqrt{100}+\sqrt{99}} \frac{(\sqrt{100}-\sqrt{99})}{(\sqrt{100}-\sqrt{99})} = \frac{\sqrt{100}-\sqrt{99}}{1}$

so $\sqrt{100}-\sqrt{99}+\sqrt{99}-\sqrt{98}+\sqrt{98}-\sqrt{97}+\cdots+\sqrt{3}-\sqrt{2}+\sqrt{2}-\sqrt{1} = 9$ Ans.

(6) What least no. must be multiplied to 1080 to make it a perfect square no. & perfect cube no.?

→ To make perfect square no.

↓

$2\times2 \times 3\times3 \times 2\times\textcircled{2} \times 3\times\textcircled{3} \times 5\times\textcircled{5}$

Ans. → $2\times3\times5 = \boxed{30}$

2	1080
2	540
2	270
3	135
3	45
3	15
	5

To make perfect cube no.

↓

$2\times2\times2 \times 3\times3\times3 \times 5\times\textcircled{5}\times\textcircled{5}$

Ans → $5\times5 = \boxed{25}$

---

(7) What least no. must be added and must be subtracted to 1080 to make it a perfect square no.?

→ $32^2 = 1024$ → 1080 − 1024 = 56 must be subtracted.

$33^2 = 1089$ → 1089 − 1080 = 9 must be added.

Or

$$\begin{array}{r|l} & 32 \\ 3 & 1080 \\ & 9 \\ \hline 62 & 180 \\ & 124 \\ \hline & 56 \end{array}$$

56 must be subtracted.

$$\begin{array}{r|l} & 33 \\ 3 & 1080 \\ & 9 \\ \hline 63 & 180 \\ & 189 \\ \hline & 9 \end{array}$$

9 must be added.

---

(8)

$$\sqrt[3]{\left[8\sqrt{\left(5\sqrt{\left(\sqrt[6]{3^4}\right)^{10}}\right)^{12}}\right]^6}$$

↓

$3^{\frac{\text{Power}}{\text{surds}}} \rightarrow 3^{\frac{4\times10\times12\times6}{6\times5\times8\times3}} = \boxed{3^4}$ Ans.

$$\sqrt[12]{\left[8\sqrt{\left(10\sqrt{\left(\sqrt[6]{5^{10}}\right)^4}\right)^{12}}\right]^6}$$

$5^{\frac{10\times4\times12\times6}{6\times10\times8\times12}} = 5^{\frac{1}{2}} = \sqrt{5}$

(9) $\sqrt{x}+\sqrt{y}$ ⊕ $\frac{1}{\sqrt{x}+\sqrt{y}}$ → Ans = $2\sqrt{x}$ –

$\sqrt{x}\pm\sqrt{y}$ ⊖ $\frac{1}{\sqrt{x}\pm\sqrt{y}}$ → Ans. $\pm 2\sqrt{y}$

ex:- $\sqrt{3}+\sqrt{2}+\frac{1}{\sqrt{3}+\sqrt{2}}=?\rightarrow\sqrt{3}+\sqrt{2}+\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}-\sqrt{2})}\rightarrow\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}$

$= 2\sqrt{3}$

or $2\sqrt{x}\rightarrow 2\times\sqrt{3}=\boxed{2\sqrt{3}}$

→ $3+2\sqrt{2}+\frac{1}{3+2\sqrt{2}}\rightarrow 2\sqrt{x}\rightarrow 2\times 3=\boxed{6}$

→ $\sqrt{3}-\sqrt{2}-\frac{1}{\sqrt{3}-\sqrt{2}}\Rightarrow -2\sqrt{y}=\boxed{-2\sqrt{2}}$ Ans.

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(10) Find the bigger & smaller among these:-

$\sqrt{7}-\sqrt{5}$, $\sqrt{5}-\sqrt{3}$, $\sqrt{3}-\sqrt{1}$, $\sqrt{17}-\sqrt{15}$ [Difference of all same]

→ Direct → $\sqrt{5}-\sqrt{3}$ → Biggest [smaller numbers]

$\sqrt{7}-\sqrt{15}$ → smallest [bigger numbers]

OR $\frac{1}{\sqrt{7}-\sqrt{5}}\rightarrow\frac{\sqrt{7}+\sqrt{5}}{7-5}$ $\frac{\sqrt{7}+\sqrt{5}}{2}$ so $\sqrt{7}-\sqrt{5}=\frac{2}{\sqrt{7}+\sqrt{5}}$

similarly all:- $\frac{2}{\sqrt{7}+\sqrt{5}}, \frac{2}{\sqrt{5}+\sqrt{3}}, \frac{2}{\sqrt{3}+\sqrt{1}}, \frac{2}{\sqrt{17}+\sqrt{15}}$

Numerator of all numbers is same so the one whose denominator is smaller is actually the bigger.

(11) if $a=999$ then find $\sqrt[3]{a(a^2+3a+3)+1}$?

→ $\sqrt[3]{a^3+3a^2+3a+1^3}\rightarrow\sqrt[3]{a^3+1^3+3\cdot1^2\cdot a+3\cdot1\cdot a^2}$

→ $\sqrt[3]{(a+1)^3}\rightarrow a+1\Rightarrow 999+1=\boxed{1000}$ Ans.

Q16 There are some cows and some hens. if Number of heads is 142 and number of feet is 392, find the number of cows & hens?

→ Cows x, Hens y

$x + y = 142$

$4x + 2y = 392$

Solve for x & y

Or Shortcut :-

No. of Animals (with 4 feet) $= \frac{\text{No. of feet}}{2} - \text{No. of heads}$

No. of Cows $= \frac{392}{2} - 142 = \boxed{54}$ Ans.

No. of Hens $= 142 - 54 = \boxed{88}$ Ans.

Q17 (1) $\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \cancel{\frac{1}{3\times4\times5}} + \frac{1}{4\times5\times6} = ?$

→ $\frac{120+30+12+6}{1\times2\times3\times4\times5\times6} = \frac{168}{24\times30} = \boxed{\frac{7}{30}}$ Ans.

(2) $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \text{-----} \frac{1}{100\times101} = ?$

Shortcut :- $\frac{\text{No. of terms}}{1\times101} = \boxed{\frac{100}{101}}$ Ans.

(3) $\frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} = ?$

$\frac{1}{3\times5} + \frac{1}{5\times7} + \frac{1}{7\times9} + \frac{1}{9\times11}$ → $\frac{4}{3\times11} = \boxed{\frac{4}{33}}$ Ans.

(4) $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\text{----}\left(2-\frac{999}{1001}\right) = ?$

→ $\frac{5}{3}\times\frac{7}{5}\times\frac{9}{7}\times\text{----}\frac{1003}{1001}$ → $\boxed{\frac{1003}{3}}$ Ans.

SURDS & INDICES

* Laws of Indices

1) $a^m \times a^n \rightarrow a^{m+n}$

2) $\frac{a^m}{a^n} = a^{m-n}$

3) $(a^m)^n \rightarrow a^{mn}$

4) $a^{mn} \rightarrow a^{m \times m \times \text{--- n times}} \neq a^{mn}$

5) $(ab)^n \rightarrow a^n b^n$, $\left(\frac{a}{b}\right)^n \rightarrow \frac{a^n}{b^n}$

6) $(-a)^n \rightarrow a^n \rightarrow n = \text{even}$
$-a^n \rightarrow n = \text{odd}$

7) $\frac{1}{a^n} = a^{-n}$

8) $a^m = a^n$ then $m=n$ or $a^m = b^m$ then $a=b$

* Laws of Surds

1) $\sqrt[n]{a^n} \rightarrow a$

2) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

3) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

4) $(\sqrt[n]{a})^m = (a)^{m/n}$

5) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

* Rationalisation

* $\sqrt{n+\sqrt{n+\sqrt{n+\cdots\infty}}} \rightarrow n(n+1)$ $\qquad$ $\sqrt{n-\sqrt{n-\sqrt{n-\cdots\infty}}} \rightarrow n(n+1)$

$\sqrt{n\sqrt{n\sqrt{n\cdots\infty}}} \in {}^{\downarrow}n$ $\qquad$ $\sqrt{n\sqrt{n\sqrt{n}}} \rightarrow \left[n^{\frac{2^n-1}{2^n}}\right]$ Ans.

* only similar surds can be added/subtracted.

* To find largest surd (taking LCM)

$\sqrt[4]{4}, \sqrt[3]{3}, \sqrt{2} \rightarrow \left(\sqrt[4]{4}\right)^{12} \rightarrow \left(4^{\frac{1}{4}}\right)^{12} \rightarrow 4^3 \rightarrow 64$

$\left(\sqrt[3]{3}\right)^{12} \rightarrow \left(3^{\frac{1}{3}}\right)^{12} \rightarrow 3^4 \rightarrow 81 \rightarrow$ largest

$\left(\sqrt{2}\right)^{12} \rightarrow \left(2^{\frac{1}{2}}\right)^{12} \rightarrow 2^6 \rightarrow 64$

* Rationalisation of $\frac{1}{\sqrt{a}+\sqrt{b}}$ is $\sqrt{a}-\sqrt{b}$ and of $\frac{1}{\sqrt{a}-\sqrt{b}} = \sqrt{a}+\sqrt{b}$

(if $a-b=1$ or a, b are consecutive)

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→ Rationalize → $\frac{1}{\sqrt{3}+\sqrt{2}} \rightarrow \sqrt{3}-\sqrt{2}$

$\frac{1}{\sqrt{5}-\sqrt{2}} \rightarrow \frac{\sqrt{5}+\sqrt{2}}{3}$

$\frac{1}{\sqrt{12}+\sqrt{5}} \rightarrow \frac{\sqrt{12}-\sqrt{5}}{7}$

* $\sqrt{n+\sqrt{n-\sqrt{n+\sqrt{n}}}\cdots\infty}$ → $\frac{\sqrt{4n-3}+1}{2}$ Ans.

$\sqrt{n-\sqrt{n+\sqrt{n-\sqrt{n}}}\cdots\infty}$ → $\frac{\sqrt{4n-3}-1}{2}$ Ans.

① if $\sqrt{x} = \frac{12.3}{123}$ then $x = ?$ (0.01)

② $\sqrt{\frac{3+\sqrt{8}}{3-\sqrt{8}}} = ?$ $(3+2\sqrt{2})$

③ $\frac{4^{10+n} \cdot 16^{3n-4}}{4^{7n}} = ?$ $\left[4^{10+n+6n-8-7n} = 4^2\right]$ (16)

④ Find largest :- $\sqrt[4]{4}, \sqrt[3]{3}, \sqrt{2}$ $(\sqrt[3]{3})$

⑤ Find smallest :- $\sqrt{3}, \sqrt[3]{2}, \sqrt{2}, \sqrt[3]{4}$ $(\sqrt[3]{2})$

⑥ $\frac{\sqrt[7]{\sqrt[5]{(21^7)^5}}}{\sqrt[5]{\sqrt[3]{(7^5)^3}}} = ? \rightarrow$ [Short cut → $\frac{21^{\frac{\text{Powers}}{\text{surds}}}}{7^{\frac{\text{Powers}}{\text{surds}}}} = \frac{21^{\frac{7\times5}{5\times7}}}{7^{\frac{5\times3}{3\times5}}} = \frac{21}{7}^{23}$] (3)

⑦ $(-3)^{(-2)(-2)(-1/4)} = ?$ $\left[\left(-\frac{1}{3}\right)^{+2} \rightarrow \left(\frac{1}{9}\right)^{(-2)^{(-1/4)}} \rightarrow (9)^{2^{(-1/4)}}\right.$
$\left.(81)^{-1/4} = \left(\frac{1}{81}\right)^{1/4} \rightarrow \left(\frac{1}{3^4}\right)^{1/4} \rightarrow \frac{1}{3}\right]$ $(\frac{1}{3})$

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⑧ $\sqrt[5]{2x-7} - 3 = 0$ then $x = ?$ $\left[\left(\sqrt[5]{2x-7}\right)^5 = 3^5 = 243\right.$
$\left.2x-7 = 243, 2x = 150\right]$ (125)

⑨ $\frac{1}{1+a^{x-y}} + \frac{1}{1+a^{y-x}} = ?$ $\left[\frac{1}{1+\frac{a^x}{a^y}} + \frac{1}{1+\frac{a^y}{a^x}} = \frac{a^x+a^y}{a^y+a^x}\right]$ (1)
(LCM) (LCM)

(10) $\left[\left\{\left(3^{1-\frac{1}{2}}\right)^{1-\frac{1}{3}}\right\}^{1-\frac{1}{4}}\right]^{\cdots n \text{ terms}} = ? \quad \left[3^{\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\cdots\frac{n}{n+1}}\right] \quad \left[(3)^{\frac{1}{n+1}}\right]$

(11) $\frac{\sqrt{n+2}+\sqrt{n-2}}{\sqrt{n+2}-\sqrt{n-2}} = \frac{3}{2}$ then $6n = ?$ (cross multiplying $5\sqrt{n-2} = \sqrt{n+2}$, $25(n-2) = (n+2)$, $n = 13$) (13)

(12) $a^{1/3} = 11$ then $a^2 - 331a = ?$ $[a(a-331) \to 1331(1331-331) \to 1331000]$ (1331000)

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(13) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{10}{3}$ and $x+y=10$ then $xy = ?$ (9)

$\left[\frac{x+y}{\sqrt{xy}} = \frac{10}{3} \to \frac{10}{\sqrt{xy}} = \frac{10}{3} \to xy = 9\right]$

(14) $3 + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}+3} + \frac{1}{\sqrt{3}-3} = ?$ (3)

$\left[3 + \frac{1}{\sqrt{3}} + \left[\frac{1}{3+\sqrt{3}} - \frac{1}{3-\sqrt{3}}\right] \to 3 + \frac{1}{\sqrt{3}} + \left[\frac{3-\sqrt{3}-3-\sqrt{3}}{9-3} = \frac{-2\sqrt{3}}{6} = \frac{-1}{\sqrt{3}}\right)\right.$

$\left. \to 3 + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} = 3\right]$

(15) $\frac{\sqrt{72}\times\sqrt{363}\times\sqrt{175}}{\sqrt{32}\times\sqrt{147}\times\sqrt{252}} = ? \quad \left[\frac{6\sqrt{2}\times 11\sqrt{3}\times 5\sqrt{7}}{4\sqrt{2}\times 7\sqrt{3}\times 6\sqrt{7}} = \frac{6\times 11\times 5}{4\times 7\times 6} = \frac{55}{28}\right]$ $\left(\frac{55}{28}\right)$

(16) $\left(x^{b+c}\right)^{b-c}\cdot\left(x^{c+a}\right)^{c-a}\cdot\left(x^{a+b}\right)^{a-b} = ?$ (1)

(17) $5\sqrt{5}\times 5^3 \div 5^{-3/2} = 5^{x+2}$, $x = ?$ $\left[5^{1+\frac{1}{2}+3+\frac{3}{2}} = 5^6 \to 5^{a+2}\right]$ (4)

(18) if $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ then $(x+y) = ?$ (8)

[Rationalising → $4+\sqrt{15}$ $\quad 4-\sqrt{15}$ → $4+\sqrt{15}+4-\sqrt{15} = 8$] or LCM

(19) Which is the least :- $(0.5)^2$, $\sqrt{0.49}$, $\sqrt[3]{0.008}$, 0.23 ? ($\sqrt[3]{0.008}$)

(20) if $\sqrt{3} = 1.732$ then $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}} = ?$ (1.732)

$\frac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}} = \frac{\sqrt{3}(\sqrt{3}+\sqrt{2})}{\sqrt{3}+\sqrt{2}} = \sqrt{3} = 1.732$

(21) $\sqrt{20+\sqrt{20+\sqrt{20}}\cdots\infty} = ?$ (5)

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(22) $\sqrt{6\sqrt{6\sqrt{6\sqrt{6\sqrt{6}}}}} = ?$ ($6^{31/32}$)

(23) $\sqrt{12+\sqrt{12}\cdots\infty} \div \sqrt{12-\sqrt{12}\cdots\infty}$ (3)

(24) which is largest :- $\sqrt{5}$, $\sqrt[3]{4}$, $\sqrt[5]{2}$, $\sqrt[7]{3}$ ($\sqrt{5}$)

L.C.M. = 210

$5^{\frac{1}{2}} = 5^{\frac{105}{210}}$, $4^{\frac{1}{3}} = 4^{\frac{70}{210}}$

(5^{105}), $(3)^{70}$, $(2)^{42}$, $(3)^{30}$

(25) Largest :- $\sqrt[3]{4}$, $\sqrt{2}$, $\sqrt[6]{3}$, $\sqrt[4]{5}$? $[(4^4), (2^6), (3^2), (5^3)]$ ($\sqrt[3]{4}$)

largest

(26) $\sqrt[12]{8\sqrt[10]{10\sqrt[6]{\left(\sqrt[10]{5^{10}}\right)^4}}}^{12}$ → $5^{\frac{10\times4\times12\times6}{6\times10\times8\times12}}$

→ $5^{\frac{1}{2}} = \sqrt{5}$ Ans

(29)

① if $3^n - 3^{n-1} = 18$ then $n^2 = ?$ $\left[3^n\left(1-3^{-1}\right)=18,\ 3^n\left(\frac{2}{3}\right)=18\right]$ (9)

$3^n = 27 = 3^3 \rightarrow n = 3$

② if $a^n = b^y = c^z$ and $b^2 = ac$ then $y = ?$ $\left(\frac{2xz}{x+z}\right)$

$[a = k^{1/x},\ b = k^{1/y},\ c = k^{1/z} \rightarrow b^2 = ac \rightarrow k^{2/y} = k^{\frac{1}{x}+\frac{1}{z}}$

$\frac{2}{y} = \frac{x+z}{xz} \rightarrow y = \frac{2xz}{x+z}$]

③ $a^x = b,\ b^y = c,\ c^z = a$ then $xyz = ?$ (1)

$[a = c^z = b^{yz} = a^{xyz} \rightarrow a^1 \rightarrow xyz = 1]$

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④ find sq. root of $7 + 2\sqrt{10}$? $\left[(\sqrt{5}+\sqrt{2})^2 \rightarrow \sqrt{5}+\sqrt{2}\right]$ $(\sqrt{5}+\sqrt{2})$

⑤ $\frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}} = ?$ (5)

[rationalising $\rightarrow \sqrt{9}+\sqrt{8}$ so $\sqrt{9}+\sqrt{8}-(\sqrt{8}+\sqrt{7})+\sqrt{7}+\sqrt{6}-(\sqrt{6}+\sqrt{5})+\sqrt{5}+\sqrt{4}$

$= \sqrt{9}+\sqrt{4} = 5$]

⑥ $2^{n+1} + 2^{n+3} = 2560$ then $n = ?$ $\left[2^n(2+2^3) \rightarrow 2^n(10)\right]$ (8)

$2^n = \frac{2560}{10} \rightarrow n = 8$

⑦ which is least:- $\sqrt{8}-\sqrt{7},\ \sqrt{7}-\sqrt{6},\ \sqrt{6}-\sqrt{5}$ - $(\sqrt{8}-\sqrt{7})$

⑧ which is greatest:- $\sqrt{19}-\sqrt{17},\ \sqrt{13}-\sqrt{11},\ \sqrt{7}-\sqrt{5}$ $(\sqrt{7}-\sqrt{5})$

[$\frac{1}{\sqrt{8}-\sqrt{7}} = \sqrt{8}+\sqrt{7}$ so $\sqrt{8}-\sqrt{7} = \frac{1}{\sqrt{8}+\sqrt{7}}$ greater the denominator lesser the no.]

(9) $\frac{1}{1+\sqrt{1}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{8}+\sqrt{9}} = ?$ (2)

Rationalise each.

$\sqrt{2}-1+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3} \cdots + \sqrt{8}-\sqrt{7}+\sqrt{9}-\sqrt{8} \rightarrow \sqrt{9}-1 = 2$

last − 1st

$\rightarrow \sqrt{9}-1 = 2$

(18) $(28+10\sqrt{3})^{1/2} - (7-4\sqrt{3})^{-1/2} = ?$ (3)

$28+2\times 5\sqrt{3} \rightarrow [(5+\sqrt{3})^2]^{1/2} - [(2-\sqrt{3})^2]^{-1/2} \rightarrow 5+\sqrt{3} - \frac{1}{2-\sqrt{3}} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$

$2(5+3)$

$4+3-2\cdot 2\sqrt{3} \rightarrow$

$5+\sqrt{3}-(2+\sqrt{3}) = 5-2 = 3$

(11) $2^x = 3^y = 6^{-z}$ $\therefore k$ then $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) = ?$ (0)

$2 = k^{1/x}$

$3 = k^{1/y}$ $\rightarrow 2\times 3 = 6 \rightarrow k^{\frac{1}{x}+\frac{1}{y}} = k^{-\frac{1}{2}} \rightarrow \frac{1}{x}+\frac{1}{y} = -\frac{1}{z} \rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$

$6 = k^{-1/z}$

(12) Given that $\sqrt[3]{3^x} = 5^{1/4}$ and $\sqrt[4]{5^y} = \sqrt{3}$ then $2xy = ?$ (3)

~~$3^{x/3} = 5^{1/4}$ and $5^{y/4} = 3^{1/2} \rightarrow 5$~~

$5^{1/4} = 3^{x/3} - ①$ $5^{y/4} = 3^{1/2}$

so $5^{1/4} = 3^{1/2y} - ②$

$① = ②$

$3^{x/3} = 3^{1/2y} \rightarrow \frac{x}{3} = \frac{1}{2y} \rightarrow 2xy = 3$

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(13) $\sqrt{20}+\sqrt{12}+\sqrt[3]{729} - \frac{4}{\sqrt{5}-\sqrt{3}} - \sqrt{81} = ?$ (0)

$2\sqrt{5}+2\sqrt{3}+9-\frac{4(\sqrt{5}+\sqrt{3})}{2}-9 \rightarrow 2\sqrt{5}+2\sqrt{3}-2\sqrt{5}-2\sqrt{3} = 0$

after rationalisation.

(14) $\frac{1}{1+\sqrt{1}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{120}+\sqrt{121}} = ?$ (10)

$\sqrt{1}-1+\sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3} \cdots \sqrt{121}-\sqrt{120} = \sqrt{121}-1 = 10$

last − 1st $\rightarrow \sqrt{121}-1 = 10$

(15) $\frac{9^n 3^2 3^n - (27)^n}{3^{3m} 2^3} = \frac{1}{27}$ then $(m-n) = ?$ (1)

$\frac{3^{2n} \cdot 9 - 3^{3n}}{3^{3m} \cdot 8} = 3^{-3} \rightarrow \frac{3^{3n}(9-1)}{3^{3m} \cdot 8} = 3^{-3} \rightarrow 3^{3n-3m} = 3^{-3}$

$3n-3m=-3$

$n-m=-1$ so $m-n=1$

(16) if $\sqrt{2} = 1.414$ then Sq. root of $\frac{\sqrt{2}-1}{\sqrt{2}+1} = ?$ (0.414)

Rationalising we get $(\sqrt{2}-1)^2$ so $\sqrt{(\sqrt{2}-1)^2} = \sqrt{2}-1 = 1.414-1 = 0.414$

(17) $\sqrt{7\sqrt{7\sqrt{7\sqrt{7}\cdots}}} \infty = (343)^{y-1}$ then $y = ?$ $7 = 7^{3y-3}$, $3y-3=1 \rightarrow y=\frac{4}{3}$ ($4/3$)

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(18) if $x = 2+\sqrt{3}$ then $\sqrt{x} + \frac{1}{\sqrt{x}} = ?$ ($\sqrt{6}$)

$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} + 2 = 2+\sqrt{3} + \frac{1}{2+\sqrt{3}} + 2 \rightarrow 2+\sqrt{3}+2-\sqrt{3}+2 = 6$

(19) if $P = \sqrt{7+\sqrt{7+\sqrt{7}\cdots}} \infty$ & $Q = \sqrt{7-\sqrt{7-\sqrt{7}\cdots}} \infty$

Find Relation b/w P & Q? ($P=-Q$, $P-Q=1$) always

$P^2 = 7+P \rightarrow P^2-P=7$

$Q^2 = 7-Q \rightarrow Q^2+Q=7$

$\rightarrow P^2-P-Q^2-Q=0$

$(P+Q)(P-Q)-1(P+Q)=0$

$\rightarrow (P+Q)(P-Q-1)=0$

$P=-Q$, $P-Q=1$

(20) $\sqrt{1+\sqrt{1+\sqrt{1}\cdots}} \infty$

$x^2 = 1+x \rightarrow x^2-x-1=0$

$\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$ ✗ → sum can't be -ve.

Find n

SIMPLIFICATION

① $3463 \times 295 - 18611 = n + 5883$

a) 997091 b) 887071 c) 989090 d) 899060 e) None

$[3463 \times 300 - 5) - (18611 + 5883) = n$
$1038900 - (17315 + 24494) = 997091]$

② $(8)^3 \div (16)^2 \times 32 = 2^{n-4} \div 4^2$

a) 12 b) 18 c) 14 d) 10 e) None

$[2^{9-8+5+4} = 2^{n-4} \rightarrow n = 14]$

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③ $[(3\sqrt{8} + \sqrt{8}) \times (8\sqrt{8} + 7\sqrt{8})] - 98 = n$

a) $3\sqrt{8}$ b) $7\sqrt{8}$ c) 382 d) 475 e) None

$[(4\sqrt{8} \times 15\sqrt{8}) - 98 \rightarrow 60 \times 8 - 98 = 480 - 98 = 382]$

④ $\sqrt{11449} \times \sqrt{6241} - (54)^2 = \sqrt{n} + (74)^2$

a) 3846 b) 3721 c) 3581 d) 3938 e) None

$[(107 \times 79) - (2916 + 5476)$
Pattern $8453 - 8392 = 61 \rightarrow n = (61)^2 = 3721]$

⑤ $\frac{3}{19}$ of 30% of 3420 $= (n)^2 \times 2$

a) $(81)^2$ b) 7 c) 9 d) 81 e) 49

$\left[\frac{3}{19} \times \frac{30}{100} \times \frac{3420}{2} = n^2 \rightarrow n^2 = 81 \rightarrow n = 9\right]$

⑥ $\sqrt{7^2 \times 24 \times 2 - (11)^3 + 3} = n$

a) 42 b) 1024 c) 1764 d) $(1024)^2$ e) 32

$\sqrt{49 \times 48 - 1328} = \sqrt{2352 - 1328} = \sqrt{1024} = 32$

$(49(50-2))$

⑦ $(0.81)^2 \div (0.729)^3 \times (0.9)^2 = (0.9)^{n-3}$

a) 6 b) 2 c) 4 d) 0 e) None

$0.9^{4-9+2} = 0.9^{n-3} \rightarrow n-3 = -3 \rightarrow n = 0$

⑧ 65% of $\sqrt{3136} \times 5 = n + 154$

a) 56 b) 28 c) 35 d) 32 e) None

$\left(\frac{65}{100} \times \frac{14}{56} \times 56\right) - 154 \rightarrow 182 - 154 = 28$

⑨ $\frac{9 \div 2 \times 27 \div 9}{18 \div 7.5 \times 5 \div 4} = n$

a) 4.5 b) 5.7 c) 2.5 d) 3.5 e) 6.8

$\frac{\frac{9}{2} \times \frac{27}{9}}{\frac{18}{375} \times \frac{50}{4}} \rightarrow \frac{27}{2 \times 3} = 4.5$

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⑩ n% of 280 + 18% of 550 = 143.8

a) 11 b) 18 c) 21 d) 16 e) None

$\frac{280n + 550 \times 18}{100} = 143.8 \times 100 \rightarrow 280n = 14380 - 9900 = 4480$

$n = 4480/280 = 16$

(11) $\sqrt{\sqrt{2500} + \sqrt{961}} = (x)^2$

a) 81 b) 3 c) 6561 d) 9 e) None

$\left[\sqrt{50+31} = \sqrt{81} \rightarrow x^2 = 9 \rightarrow x = 3\right]$

(12) $1\frac{4}{7} + 1\frac{3}{5} + 1\frac{1}{3} = x$

a) $5\frac{47}{105}$ b) $4\frac{58}{105}$ c) $4\frac{53}{105}$ d) $5\frac{43}{105}$ e) None

$\left[(1+1+1) + \left(\frac{4}{7}+\frac{3}{5}+\frac{1}{3}\right) \rightarrow 3 + \frac{60+63+35}{105} = 3 + \frac{158}{105} \rightarrow 3+1\frac{53}{105}\right.$

$\left. 4\frac{53}{105}\right]$

(13) $64^{12} \div 4^{15} = 64^x$

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a) 9 b) 3 c) 12 d) 7 e) None

$\left[64^{12-5} = 64^x \rightarrow x = 7\right]$

(14) 14% of 80 + x% of 90 = 31.9

a) 16 b) 23 c) 18 d) 26 e) None

$\left[\frac{1120 + 90x}{100} = 31.9 \times 10 \rightarrow 90x = 3190 - 1120 = 2070\right.$

$\left. x = 23\right]$

(15) $3\frac{6}{7} - 6\frac{1}{4} + 5\frac{1}{3} = x$

$-(6 + \frac{1}{4}) \rightarrow -6 - \frac{1}{4}$

a) $1\frac{65}{84}$ b) $8\frac{1}{84}$ c) $2\frac{79}{84}$ d) $5\frac{47}{84}$ e) None

$\left[(3-6+5) + \left(\frac{6}{7} - \frac{1}{4} + \frac{1}{3}\right) \rightarrow 2 + \frac{72-21+28}{84} = 2 + \frac{79}{84} = 2\frac{79}{84}\right]$

(16) $\frac{1}{6}$ of 92% of $1\frac{1}{23}$ of 650 = 85 + x

a) 18 b) 21 c) 19 d) 28 e) None

$\left[\left(\frac{1}{6} \times \frac{92^{4}}{100_{2}} \times \frac{24}{23} \times \frac{13}{650}\right) - 85 \rightarrow 104 - 85 = 19\right]$

(17) $92 \times 576 \div \left(2\sqrt{1296}\right) = (x)^3 + \sqrt{49}$

a) 3 b) $(9)^2$ c) 9 d) 27 e) None

$\left[\frac{92 \times 576^{32}}{2 \times 36_{18}} - 7 \rightarrow 736 - 7 = 729 = x^3 \rightarrow x = 9\right]$

(18) $3\frac{1}{4} + 2\frac{1}{2} - 1\frac{5}{6} = \frac{x^2}{10} + 1\frac{5}{12}$

a) 25 b) $\sqrt{5}$ c) 625 d) 15 e) 5

$\left[(3+2-1-1) + \left(\frac{1}{4} + \frac{1}{2} - \frac{5}{6} - \frac{5}{12}\right) = \frac{x^2}{10}\right.$

$\left.3 + \frac{3+6-10-5}{12} \rightarrow 3 - \frac{1}{2} \rightarrow \frac{5}{2} = \frac{x^2}{10} \rightarrow x = 5\right]$

(19) $\left(\sqrt{8} \times \sqrt{8}\right)^{\frac{1}{2}} + (9)^{\frac{1}{2}} = (x)^3 + \sqrt{8} - 340$

a) 7 b) 19 c) 18 d) 9 e) None

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$\left[8^{\frac{1}{2}} + 3 - 8^{\frac{1}{2}} + 340 \rightarrow 343 = x^3 \rightarrow x = 7\right]$

(20) $(15 \times 0.40)^4 \div (1080 \div 30)^4 \times (27 \times 8)^4 = (3 \times 2)^{x+5}$

a) 8 b) 3 c) 12 d) 16 e) None

$\left[6^4 \div 36^4 \times (216)^4 = 6^{x+5}\right.$

$\left.6^{4-8+12} = 6^{x+5} \rightarrow x + 5 = 8 \rightarrow x = 3\right]$

NUMBER SERIES

Basic Types of Series Pattern :-

1) Addition / subtraction Pattern (most Imp.)
2) Multiplication / Division Pattern.
3) Multiplication & addition Mix Pattern.
4) Squares - Cubes Pattern.

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* MISSING NO. SERIES / NEXT NO. SERIES

A Addition/subtraction Pattern :- This Pattern will be followed if the gap between Ist & last term of series is less. (under 1000)

① 155, 151, 139, 119, 91, ____ → 80 → 91 − 36 = 55 Ans.
(4, 12, 20, 28, 36)

② 445, 444, 436, 409, 345, ____ → 345 − 125 = 220 Ans.
(1, 8, 27, 64, 125 → 5^3)

③ 759, 423, 255, 171, 129, ____ → 129 − 21 = 108 Ans.
(336, 168, 84, 42, 21 = 42 ÷ 2)

④ 0, 6, 24, 60, 120, 210, ____ → 210 + 126 = 336 Ans.
(6, 18, 36, 60, 90, 126)
(12, 18, 24, 30 → 36)
2 tier addition

⑤ 144, 173, 140, 169, 136, ____ 165 Ans.
(+4, −4, −4, +4)
Inc. - Dec series Mix

⑥ 13, 20, 39, 78, 145, ____ → 145 + 103 = 248 Ans.
(7, 19, 39, 67, 103)
(12, 20, 28, 36)

⑦ 8, 8, 9, 9, 11, 10, 14, 11, ____ 18 Ans.
(+1, +2, +3, +4)
Mix series of two

⑧ 3, 100, 297, 594, 991, ____ 991 + 497 = 1488 Ans.
(97, 197, 297, 397, 497)

⑨ 144, 132, 125, 113, 105, 93, ____ 93 − 9 = 84 Ans.
(12, 7, 12, 8, 12, 9)

⑩ 80, 76.5, 69.5, 59, 45, 27.5, ____ 27.5 − 21 = 6.5 Ans.
(3.5, 7, 10.5, 14, 17.5, 21)

B Square – Cubes Pattern :-

① 2, 9, 28, 65, ——, 217, 344 → 126 Ans.

[1^3+1, 2^3+1, 3^3+1, 4^3+1, 5^3+1, ---]

② 1, 1, 4, 8, 9, 27, ——, 64 → 16 Ans.

[1^2, 1^3, 2^2, 2^3, 3^2, 3^3, 4^2, 4^3]

③ 8, 12, 39, 55, 180, —— → 180+36 = 216 Ans.

4, 27, 16, 125, 36

[2^2, 3^3, 4^2, 5^3, 6^2]

④ −2, 4, 22, 58, 118, 208, —— → 334 Ans.

[1^3-3, 2^3-4, 3^3-5, 4^3-6, 5^3-7, 6^3-8, 7^3-9]

or check 2 tier addition.

⑤ 2, 3, 10, 15, 26, 35, 50, —— → 63 Ans.

[1^2+1, 2^2-1, 3^2+1, 4^2-1, 5^2+1, 6^2-1, 7^2+1, 8^2-1]

⑥ 0, 4, 18, 48, 100, 180, —— → 294 Ans.

[1^3-1^2, 2^3-2^2, 3^3-3^2, 4^3-4^2, 5^3-5^2, 6^3-6^2, 7^3-7^2]

or check 2 tier addition. SANDEEP BOHAY™

⑦ 1, 1, 1, 4, 2, 1, 9, ——, 1, 16 → 3 Ans.

[1^2, 1^1, 1^0, 2^2, 2^1, 2^0, 3^2, 3^1, 3^0, 4^2]

⑧ 895, 870, 821, 740, 619, —— 619−169 = 450 Ans.

25, 49, 81, 121, 169

5^2, 7^2, 9^2, 11^2, 13^2

C MULTIPLICATION & addition Pattern:- (check Range)

① 1, 4, 12, 30, 68, 146, ——

Range → 2–3 [Multiple is between 2 & 3 in most terms]
Constant

so 1×2+2 = 4
4×2+4 = 12
12×2+6 = 30
30×2+8 = 68
68×2+10 = 146
146×2+12 = 304 Ans.

or 2 tier addition

[Range is constant so multiple will also be constant.
most of the constant range series also follow the addition pattern]

② 7, 8, 18, 57, ——, 1165

Range → 1-2, 2-3, 3-4, 4-5 ---

Increasing

So $7 \times 1 + 1 = 8$

$8 \times 2 + 2 = 18$

$18 \times 3 + 3 = 57$

$57 \times 4 + 4 = \boxed{232}$ Ans.

[Range is Increasing so multiple will also increase.]

[Increased Range series will never follow the addition Pattern]

③ 45, 15, 6, 3, 2, 2, —— → $2 \div 0.5 = \boxed{4}$ Ans.

$\underbrace{\div 3}_{45,15}$ $\underbrace{\div 2.5}_{15,6}$ $\underbrace{\div 2}_{6,3}$ $\underbrace{\div 1.5}_{3,2}$ $\underbrace{\div 1}_{2,2}$ $\underbrace{\div 0.5}_{2,——}$

④ 15, 25, 40, 65, ——, 195

[Range → 1-2 Constant so Pattern → $\times 2 - 5$, $\times 2 - 10$, $\times 2 - 15$ -- so $65 \times 2 - 20 = \boxed{110}$ Ans.]

or check 2 tier addition

⑤ 5, 6, 16, 57, 244, ——

[Range → 1-2, 2-3, 3-4, --- increasing so Pattern $\times 1 + 1^2$, $\times 2 + 2^2$, $\times 3 + 3^2$ --- so $244 \times 5 + 5^2$ $= \boxed{1245}$ Ans.]

⑥ 16, 8, 12, 30, ——, 472.5

[$\times 0.5$, $\times 1.5$, $\times 2.5$, $\times 3.5$, $\times 4.5$] → $30 \times 3.5 = \boxed{105}$ Ans.

⑦ 32, 49, 83, 151, 287, 559, ——

[Constant Range → (1-2) → Pattern ⇒ $\times 2 - 15$ so $559 \times 2 - 15 = \boxed{1103}$ Ans.]

or check addition.

⑧ 11, 10, 18, 51, 200, ——

[$\times 1 - 1$, $\times 2 - 2$, $\times 3 - 3$, $\times 4 - 4$, $\times 5 - 5$] → $200 \times 5 - 5 = \boxed{995}$ Ans.

⑨ 824, 408, 200, 96, 44, 18, ——

[$\times 2 + 8$ Pattern from Right Side] → (5) $\times 2 + 8 = 18$ → $\boxed{5}$ Ans.

or check addition.

⑩ 6072, ——, 200, 48, 14, 5, 3

[$3 \times 1 + 2 = 5$, $5 \times 2 + 4 = 14$, $14 \times 3 + 6 = 48$, $48 \times 4 + 8 = 200$, $200 \times 5 + 10 = \boxed{1010}$ Ans.]

(11) 2185, ——, 241, 79, 25, 7, 1

[Constant Range → (3-4) Pattern → ×3+4] → 241×3+4 = 727 Ans.

or check addition.

(12) 1, 2, 4, 12, 15, 60, ——

[×2, +2, ×3, +3, ×4, +4] → 60+4 = 64 Ans.

(13) 3, 9, 36, 72, 216, 864, 1728, ——

[×3, ×4, ×2, ×3, ×4, ×2, ×3] 1728×3 = 5184 Ans.

(14) 9, 62, ——, 1854, 7415, 22244

[Decreasing Range → 6-7, 5-6, 4-5, 3-4, 2-3] → 9×7−1 = 62

62×6−1 = 371 Ans.

371×5−1 = 1854

1854×4−1 = 7415

7415×3−1 = 22244

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* WRONG NUMBER SERIES :-

It requires a little more time but pattern is same as of next no series.

(1) 5, 348, 564, 689, 723, 780, 788

343 216 125 34 57 8

7^3 6^3 5^3 4^3 3^3 2^3

You will find that two numbers are wrong. All you need to do is correct one, then the second one will automatically get correct.

→ 64 should come in place of 34 so 753 will come in place of 723 & hence next difference will be 27 in place of 57.

or 723 lies in between 34 & 57 so it is wrong term.

② 2, 3, 7, 26, 121, 721, 5041

[Inc. Range → 1-2, 2-3, 3-4, -- So $2\times2-1=3$, $3\times3-2=7$, $7\times4-3=\boxed{25}$ so 26 is wrong.]

③ 3, 9, 18, 54, 110, 324, 648

(×3, ×2, ×3, ×2, ×3, ×2)

[$54\times2=\boxed{108}$ so 110 is wrong]

④ 2, 6, 12, 27, 58, 121, 248

[$2\times2+1=\boxed{5}$, $5\times2+2=12$, $12\times2+3=27$, $27\times2+4=58$] so 6 is wrong

⑤ 2, 9, 28, 65, 126, 216, 344

[$1^3+1, 2^3+1, 3^3+1, 4^3+1, 5^3+1, 6^3+1=\boxed{217}$ will come in place of 216]

⑥ 5531, 5506, 5425, 5304, 5135, 4910, 4621

9^2, 11^2, 13^2, 15^2, 17^2, $\times(5^2 \to 7^2)$

so 5531 is wrong 5555 will come here.

⑦ 325, 259, 204, 160, 125, 105, 94

66, 55, 44, 35, 20, 11

33^2, 22^2

so 125 is wrong, 127 will come here.

NEXT NO: SERIES PRACTICE

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		Ans
①	18, 96, 161, 213, ——, 278	252
②	7, 13, 24, 40, 61, ——	87
③	25, 48, 94, 186, 370, ——	738
④	12, 35, 81, 173, ——, 725	357
⑤	1, 2, 7, 34, 203, ——	1420
⑥	5, 8, 22, ——, 124, 246, 736	42

⑦ 1, 3, 10, 29, 74, ___, 382 — 173

⑧ 45, 46, 70, 141, ___, 1061.5 — 353.5

⑨ 620, 632, 608, 644, 596, ___ — 656

⑩ 120, 320, ___, 2070, 5195, 13007.5 — 820

⑪ 117, 389, 525, ___, 627 — 593

⑫ 1050, 420, 168, 67.2, 26.88, 10.752, ___ — 4.3008

⑬ 462, 552, 650, 756, 870, 992, ___ — 1122

⑭ 18935, ___, 750, 145, 25, 2 — 3780

⑮ 16, 24, ___, 210, 945, 5197.5 — 60

⑯ 142, 871, 1383, 1726, 1942, 2067, ___ — 2131

⑰ 958, 833, 733, 658, 608, ___ — 583

⑱ 2, 7, 27, 107, 427, ___ — 1707

⑲ 4, 6, 9, 13.5, ___ — 20.25

⑳ 10.4, 10.65, 11.15, 11.9, 12.9, ___ — 14.15

WRONG NO. SERIES

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① 8, 5, 6.5, 11, 26, 68, 207.5 — 11

② 45, 15, 6, 3, 2, 1, 4 — 1

③ 8424, 4212, 2106, 1051, 526.5, 263.25 — 1051

④ 49, 56, 64, 71, 81, 90, 100, 110 — 71

⑤ 1, 2, 6, 12, 66, 197, 786 — 12

⑥ 1, 1.5, 3, 20.25, 121.5, 911.25, 8201.25 — 3

⑦ 2, 7, 28, 60, 126, 215, 344 — 60

⑧ 2, 3, 10, 15, 25, 35, 50, 63 — 25

⑨ 5, 7, 13, 25, 45, 87, 117 — 87

⑩ 824, 408, 396, 96, 44, 18, 5 — 396

⑪ 2807, 1400, 697, 347, 171, 84, 41, 20 — 347

⑫ 318, 368, 345, 395, 372, 422, 400, 449 — 400

LINEAR-QUADRATIC EQUATIONS

* LINEAR EQUATION → $2x+3y=19$, Degree = 1 (Highest Power)

QUADRATIC EQUATION → $x^2-7x+12=0$, Degree = 2

→ No. of Solutions depend upon degree of equation.

Degree = 1 then No. of Solutions = 1 (value of x, y etc.)

Degree = 2 then No. of Solutions = 2

NOTE!- if $x=\sqrt{144}$, then $x=12$ only. [Degree = 1]

CRAM if $x^2=144$, then $x=\pm 12$ [Degree = 2]

---

* Two Equations will be given in exam in form of x & y or p & q, you will have to find the values of these variables & then compare these values to arrive at given options!-

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(1) $x>y$ (2) $x\geq y$ (3) $x<y$ (4) $x\leq y$

(5) $x=y$ or relationship can't be estabilished.

Let us see how to choose one option out of this!-

* Let 2 solutions of x are x_1 & x_2 and 2 solutions of y are y_1 & y_2.

(1) if $x_1>y_1$, $x_1>y_2$, $x_2>y_1$, $x_2>y_2$ then $x>y$

(2) if $x_1>y_1$, $x_1>y_2$, $x_2<y_1$, $x_2>y_2$ then Can't be estb.

(3) if $x_1>y_1$, $x_1=y_2$, $x_2>y_1$, $x_2>y_2$ then $x\geq y$

(4) if $x_1<y_1$, $x_1<y_2$, $x_2=y_1$, $x_2<y_2$ then $x\leq y$

(5) $<$, $<$, $<$, $<$ then $x<y$

(6) $>$, $>$, $<$, $<$ then Can't be estb.

'=' means '>' & '<' Can't come together, if & it is, then relationship Can't be estb. (option 5)

(7) if $x=y$ or relationship can't be estb. then option (5)

① $2x+3y=19$]×4 , $7x-4y=23$]×3

$8x+12y=76$

$21x-12y=69$

$29x=145$

$\boxed{x=5}$

Putting in ① → $2(5)+3y=19$

$\boxed{y=3}$

$x=5$, $y=3$

$\boxed{x>y}$

---

② $\frac{x}{4}+\frac{y}{3}=\frac{5}{12}$, $\frac{x}{2}+y=1$

↓ ↓

$3x+4y=5$, $x+2y=2$

By solving

$x=1$, $y=\frac{1}{2}$

$\boxed{x>y}$

---

③ $x=\sqrt{3136}$, $y^2=3136$

↓ ↓

$x=56$, $y=\pm 56$

$56=56$

$56>-56$

$\boxed{x\geq y}$

---

④ $x=\sqrt[3]{729}$, $y=\sqrt{81}$

↓ ↓

$x=9$, $y=9$

$\boxed{x=y}$ → option (S)

---

⑤ $\frac{3}{\sqrt{x}}+\frac{4}{\sqrt{x}}=\sqrt{x}$, $y^2-\frac{(7)^{5/2}}{\sqrt{y}}=0$

$\frac{3+4}{\sqrt{x}}=\sqrt{x}$ ↓

$x=7$, $\frac{y^{5/2}-7^{5/2}}{\sqrt{y}}=0$

$y=7$

$\boxed{x=y}$

---

⑥ $x^2+3=12$, $3y-5=1+y$

$x^2=9$, $2y=6$

$x=+3,-3$, $y=3$

$\Rightarrow x_1=y_1$

$x_2<y_1$

$\boxed{x\leq y}$

* QUICK WAY TO SOLVE QUADRATIC EQUATION :-

Example → $x^2 - 13x + 42 = 0$

Basic method :- $x^2 - 7x - 6x + 42 = 0$

$x(x-7) - 6(x-7) = 0$

$(x-7)(x-6) = 0$

$x = 7, x = 6$

Quick method :-

$1 \cdot x^2 - 13x + 42 = 0$ (+42)

$-6, -7$

$-6 \times -7 = +42$

$-6 - 7 = -13$

$+6, +7$

Example 2 :→ $3x^2 + 17x + 10 = 0$ (+30)

$15, 2$

$\frac{15}{3}, \frac{2}{3}$

$-5, -\frac{2}{3}$

1) Multiply coefficient of x^2 (3) with constant (10) → $3 \times 10 = 30$

2) Break coeff. of x in two parts such that their product = 30.

3) Divide these by coeff of x^2 (3)

4) Change the signs.

---

① $4x^2 = 16$

$x^2 = 4$

$x = +2, -2$

$x < y$

$1 \cdot y^2 - 10y + 25 = 0$ (+25)

$-5, -5$

$5, 5$

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② $x^2 + 17x + 72 = 0$

$9, 8$

$-9, -8$

$y^2 + 15y + 56 = 0$

$8, 7$

$-8, -7$

$x \leq y$

③ $4x^2 - 5x + 1 = 0$ (+4)

$-4, -1$

$-\frac{4}{4}, -\frac{1}{4}$

$1, \frac{1}{4}$

$y^2 - 2y + 1 = 0$

$-1, -1$

$1, 1$

$1 = 1$

$1 = 1$

$\frac{1}{4} < 1$

$\frac{1}{4} < 1$

$x \leq y$

(4) $3x^2+7x-6=0$, $12y^2-17y+6=0$

-18

$9, -2$

$-\frac{9}{3}, +\frac{2}{3}$

$-3, 0.66$

$+72$

$-9, -8$

$\frac{9}{12}, \frac{8}{12}$

$0.75, 0.66$

$-3 < 0.75$

$-3 < 0.66$

$0.66 < 0.75$

$0.66 = 0.75$

$x \leq y$

(5) $8x^2+6x-5=0$, $12y^2-22y+8=0$

$\frac{10}{8}, -\frac{4}{8}$

$-\frac{5}{4}, \frac{1}{2}$

$<$

$<$

$=$

$x \leq y$

$-16, -6$

$\frac{16}{12}, \frac{6}{12}$

$\frac{4}{3}, \frac{1}{2}$

(6) $12x^2+11x+12=10x^2+22x$, $13y^2-18y+3=9y^2-10y$

$2x^2-11x+12=0$

$-\frac{8}{2}, -\frac{3}{2}$

$4, 1.5$

$>$

$=$

$>$

$x \geq y$

$4y^2-8y+3=0$

$-\frac{6}{4}, -\frac{2}{4}$

$\frac{3}{2}, 0.5$

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(7) $20x^2-33x+7=0$, $y=\sqrt{0.0625}$

$+140$

$-\frac{28}{20}, -\frac{5}{20}$

$\frac{7}{5}, \frac{1}{4}$

$1.4 \quad 0.25$

$1.4 > 0.25$

$0.25 = 0.25$

$x \geq y$

$y = \sqrt{\frac{625}{10000}} = \frac{25}{100}$

0.25

(8) $(x^{\frac{7}{5}} \div 9) = 169 \div x^{\frac{3}{5}}$, $y^{\frac{1}{4}} \times y^{\frac{1}{4}} \times 7 = 273 \div y^{\frac{1}{2}}$

$x^{\frac{7}{5}+\frac{3}{5}} = 169 \times 9$

$x^2 = 169 \times 9$

$x = +39$

-39

$=$

$<$

$x \leq y$

$y^{\frac{1}{4}+\frac{1}{4}+\frac{1}{2}} = \frac{273}{7}$

$y = 39$

(9) $6x^2+5x+1=0$, $15y^2+8y+1=0$

$\frac{3}{6}, \frac{2}{6}$ | $\frac{5}{15}, \frac{3}{15}$

$-\frac{1}{2}, -\frac{1}{3}$ | $-\frac{1}{3}, -\frac{1}{5}$

$-0.5, -0.33$ | $-0.33, -0.2$

$x < y$

(10) $88x^2-19x+1=0$, $132y^2-23y+1=0$

$\frac{-11}{88}, \frac{-8}{88}$ | $\frac{-11}{132}, \frac{-12}{132}$

$\frac{1}{8}, \frac{1}{11}$ | $\frac{1}{12}, \frac{1}{11}$

$x > y$

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(11) $10x^2-7x+1=0$, $35y^2-12y+1=0$

$\frac{-5}{10}, \frac{-2}{10}$ | $\frac{-7}{35}, \frac{-5}{35}$

$\frac{1}{2}, \frac{1}{5}$ | $\frac{1}{5}, \frac{1}{7}$

$0.5, 0.2$ | $0.2, 0.14$

$x > y$

(12) $144x^2-16=9$, $12y+\sqrt{4}=\sqrt{49}$

$144x^2=25$ | $12y=5$

$x^2=\frac{25}{144}$ | $y=\frac{5}{12}$

$x=+\frac{5}{12}, -\frac{5}{12}$

$x < y$

(13) $3x^2-19x+28=0$, $5y^2-18y+16=0$

+84 | +80

$\frac{-12}{3}, \frac{-7}{3}$ | $\frac{-10}{5}, \frac{-8}{5}$

$4, 2.33$ | $2, 1.6$

$x > y$

(14) $\frac{\sqrt{x}}{5}+\frac{3\sqrt{x}}{10}=\frac{1}{\sqrt{x}}$, $\frac{10}{\sqrt{y}}-\frac{2}{\sqrt{y}}=4\sqrt{y}$

$\frac{2\sqrt{x}+3\sqrt{x}}{10}=\frac{1}{\sqrt{x}}$ | $\frac{8}{\sqrt{y}}=4\sqrt{y}$

$5x=10$ | $4y=8$

$x=2$ | $y=2$

$x=y$

(15) $8x^2+10x+3=0$, $5y^2+19y+12=0$

$\frac{6}{8}, \frac{4}{8}$ → $-0.66, -0.5$

$\frac{15}{5}, \frac{4}{5}$ → $-3, -0.8$

$x > y$

(16) $8x^2+6x-5=0$, $17x^2+48x-9=0$

$\frac{10}{8}, -\frac{4}{8}$ → $-\frac{5}{4}, +\frac{1}{2}$ → $-1.25, 0.5$

$\frac{51}{17}, \frac{-3}{17}$ → $-3, +0.17$

Can't be estb. → Option (5)

(17) $6x^2-11x+4=0$, $13y^2-32y+12=0$

$-\frac{8}{6}, -\frac{3}{6}$ → $\frac{4}{3}, \frac{1}{2}$ → $1.33, 0.5$

$\frac{-26}{13}, \frac{-6}{13}$ → $2, \frac{6}{13}$ → $2, 0.46$

Can't be estb.

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(18) $8x^2-78x+169=0$, $20y^2-117y+169=0$

$8 = 4\times2$, $13\times13 = 169$

$13\times4=52$, $13\times2=26$

$20 = 4\times5$, $169 = 13\times13$

$13\times5=65$, $13\times4=52$

So $8x^2-78x+169=0$

$-52, -26$

$\frac{52}{8}, \frac{26}{8}$

$\frac{13}{2}, \frac{13}{4}$

$6.5, 3.25$

$20y^2-117y+169=0$

$-65, -52$

$\frac{65}{20}, \frac{52}{20}$

$\frac{13}{4}, \frac{13}{5}$

$3.25, 2.6$

$x \geq y$

RATIO

* Ratio * Partnership * Ages * Alligation

Q1 A : B : C → 2 : 3 : 5. Find the share of B out of total amount of ₹ 500.

→ $B = \frac{3}{2+3+5} \times 500 = \frac{3}{10} \times 500 = 150$

Q2 Inverse ratio

3 : 5 → Inverse → $\frac{1}{3} : \frac{1}{5}$ → $\frac{5:3}{15}$ → 5 : 3

2 : 7 → 7 : 2

2 : 3 : 5 → Inverse → 3×5 : 2×5 : 2×3 → 15 : 10 : 6

Q3 (I) A : B → 2 : 3, B : C → 3 : 5, Find A : B : C → ?

A	B	C
2	3	
	3	5

2 : 3 : 5

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(II) A : B → 3 : 4, B : C → 2 : 5, find A : B : C ?

A	B	C
3×1	4×1	
2×2	5×2	

→

A	B	C
3	4	
	4	10

3 : 4 : 10

Shortcut

A	B	C
3	4 → (4)	
× ↓ (2)	← 2	5
6	8	20

3 : 4 : 10

(III) A : B → 1 : 2, B : C → 3 : 2, C : D → 1 : 3. Find A : B : C : D ?

A	B	C	D
1×2	2×3/2		
3×1	2×1		
1×2	3×2		

3/2 : 3 : 2 : 6

3 : 6 : 4 : 12

Shortcut

A	B	C	D
1	2 → (2)	→ (2)	
× ↓ (3)	← 3	2 → (2)	
(1)	← (1)	← 1	3

3 : 6 : 4 : 12

Q4 20% of A = 30% of B = 40% of C. Find A : B : C ?

→ $\frac{20}{100}A = \frac{30}{100}B = \frac{40}{100}C$

$2A = 3B = 4C$

$\frac{A}{B} = \frac{3}{2}$ $\quad \frac{B}{C} = \frac{4}{3}$

A : B : C

3 : 2 → ②

④ ← 4 : 3

12 : 8 : 6

6 : 4 : 3

Shortcut $2A = 3B = 4C$

Reverse the digits & take ratio

A : B : C

$\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

$\frac{6 : 4 : 3}{12}$

6 : 4 : 3

Q5 A : B : C : D = 3 : 2 : 5 : 4, D : E = 3 : 5. Find A : B : C : D : E ?

→ A : B : C : D : E

3 : 2 : 5 : 4 → ④

③ ← ③ ← ③ ← 3 : 5

9 : 6 : 15 : 12 : 20

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Q6 A bag contains ₹410 in form of ₹5, ₹2 and ₹1 coins. The no. of coins are in ratio 4 : 6 : 9. Find no. of ₹2 coins?

→ No. → ₹5 : ₹2 : ₹1

4 : 6 : 9

4n, 6n, 9n → $4n \times 5 + 6n \times 2 + 9n \times 1 = 410$

$41n = 410$

$n = 10$

₹2 coins = $6 \times 10 = \boxed{60}$

Q7 A bag contains ₹55 in form of ₹1, 50P and 25P coins in the ratio 1 : 2 : 3. Find no. of 50P coins?

→ ₹1 : 50P : 25P

No. → 1 : 2 : 3

n, 2n, 3n → $n \times 1 + 2n \times \frac{1}{2} + 3n \times \frac{1}{4} = 55$

$\frac{4n + 4n + 3n}{4} = 55$

$n = 20$

50P coins = $2 \times 20 = \boxed{40}$

Q8 The no. of employees are reduced in ratio 3:2 and salary of each increased in ratio 4:5. By doing so, company saves ₹ 40,000. find initial expenditure on salary?

→

	Before		Now
No. →	3	:	2
Salary of each →	4	:	5
	12	:	10 → expenditure

exp. = No. × Salary of each

(−2)

Saving = 2 → 40,000
1 → 20,000
12 → 2,4000 → Initial expenditure.

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Q9 ₹ 355 were divided among A, B, C in such a way that A had ₹ 20 more than B and C had ₹ 15 more than A. Find share of C ?

→ A: x+20, B: x, C: x+20+15

x+20+x+x+35 = 355
3x = 300
x = 100
C = 100+35 = 135

Q10 Income of A & B → 2:3 and expenses → 5:9. if each saves ₹ 1500, then find their incomes ?

→ A B

Income − saving = expense

Incomes 2 : 3 → 2x : 3x

A/B → (2x − 1500)/(3x − 1500) = 5/9 → 18x − 13500 = 15x − 7500

3x = 6000
x = 2000

expense = 5 : 9

saving = 1500, 1500

Income of A = 2×2000 = 4000
B = 3×2000 = 6000

Shortcut

A : B
3 (2 : 3) 6
5 : 9

1500 , 1500

if both saves equal amt then diff of ratios of Income & exp. should also be equal (3 ≠ 6) for this, multiply the difference of 1st ratio with second and difference of 2nd ratio with first

A : B

2 ⟵ 3] ×4

5 ⟵ 9] ×1

→

A	:	B	
8	:	12	×3
5	:	9	
1500		1500	

3 → 1500

1 → 500

8 → 500×8 = 4000 = A

12 → 500×12 = 6000 = B

Q 11 Two numbers are in ratio 4:5. if each no. reduced by 25 then ratio becomes 3:4. Find 2nd No.?

→

I	II
4	5
↓	
3	4

$$\frac{4n-25}{5n-25} = \frac{3}{4}$$

$16n - 100 = 15n - 75$

$n = 25$

2nd No. → 5×25 = **125**

Shortcut

I	II	
4	5	−1
3	4	−1

1 → 25

5 → 125 → 2nd No.

Q 12 In a mixture of 100 ltrs, the ratio of milk & water is 2:3. Find quantity of water to be added to make this ratio 1:2 ?

→ 100 ltr

M	:	W	
2	:	3	
↓			
40		60	

ltrs → 40 60

$\frac{M}{W} \rightarrow \frac{40}{60+n} = \frac{1}{2}$

$80 = 60 + n$

$n = 20$ ltrs

Or

M	:	W	
40		60	+20
↓			
40		80	
↓			
1	:	2	

So 20 ltrs to be added to make 1:2

Or

M	:	W	
2	:	3	
↓			
1	:	2	×2 (+1)
2	:	4	

2+3 → 5 = 100 ltrs

1 = 20 ltrs

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Q 13 Price of gold is directly proportional to square of its weight. if it is broken in ratio 3:2:1, then a person faces a loss of ₹ 2800. Find initial price of gold?

→ Price ∝ (weight)²

3 + 2 + 1 → 6

$3^2 + 2^2 + 1^2 \rightarrow 6^2$

14 → 36

22 → 2200

1 → 100

36 → 3600 Ans.

Q14 Salaries of A, B and C → 1 : 3 : 4. If their salaries are increased by 5%, 10% and 15% resp, then find ratio of increased salaries?

→

A	B	C
100	300	400
↓5%	↓10%	↓15%
105	330	460

21 : 66 : 92

Q15 ₹ 425 divided among 4 men, 5 women and 6 boys such that share of a man, a woman and a boy is in ratio 9 : 8 : 4. Find share of a woman?

→ Ratio of shares → 4×9 : 5×8 : 6×4 → 4M : 5W : 6B → ~~36~~ : ~~40~~ : ~~24~~

9 : 10 : 6

5 women = $\frac{10}{25} \times 425 = 170$

1 woman = $\frac{170}{5} =$ **₹34**

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Q16 One year ago ratio of salaries of A & B was 3 : 5. Ratio of their individual salaries of last year and present year are 2 : 3 and 4 : 5 resp. If their total salaries of present year are ₹ 8600, find present salary of A?

→

A	B	
3	5	→ Last yrs ratio
$3 \times \frac{3}{2}$	$5 \times \frac{5}{4}$	→ Present year ratio
$\frac{9}{2}$	$\frac{25}{4}$	→ 18 : 25

$A = \frac{18}{43} \times 8600 =$ **3600**

Q17 Some money divided among A, B & C such that 5 times A's share, 3 times B's share and 2 times C's share are all equal. Find ratio of their shares?

→ 5A = 3B = 2C

A : B : C → $\frac{1}{5} : \frac{1}{3} : \frac{1}{2}$ → $\frac{6 : 10 : 15}{30}$ Ans.

Q18 [Partnership ⇒ Profit ∝ Investment × time]

A began a business with ₹450 and was joined afterwards by B with ₹300. After how many months did B join if the profits after one year were divided in ratio 2:1?

→ A → 45 × 12
B → 30 × x

$\frac{45 \times 12}{30 \times x} = \frac{2}{1} \rightarrow x = 9$ so after 3 months

---

Q19 The investment made by A & B are in ratio 3:2. If 5% of total profit is donated & A gets ₹8,550 as his share. What is total profit?

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→ A : B = 3 : 2

3 → 8550
1 → $\frac{8550}{3}$
5 → $\frac{8550 \times 5}{3} = 14250$

95% → 14250
100% → $\frac{100}{95} \times 14250$
= 15000

Q20 A & B entered partnership with 16000 & 12000 resp. After 3 mnths A withdrew 5000 while B invested 5000 more. After 3 mnths more, C joins with ₹21000. After one year they made a profit of ₹26,400. Find Difference b/w share of B and C?

→ A → 16×3 + 11×9 = 147 49 7
B → 12×3 + 17×9 = 189 63 9
C → 21×6 = 126 42 6

B − C = $\frac{9-6}{7+9+6} \times 26400$

$= \frac{3}{22} \times 26400 = 3600$

Q21 A, B, C invested their Capitals in ratio 5:6:8. and at the end of term, recieved their profits in ratio 5:3:12. Find the ratio of their time periods?

→ Profit = Capital × time

time = $\frac{\text{Profit}}{\text{Capital}}$ so $t_1 : t_2 : t_3 \rightarrow \frac{5}{5} : \frac{3}{6_2} : \frac{12}{8_2}$

$\frac{2:1:3}{2} \rightarrow 2:1:3$

Q22 Ratio of Ages of A and B is 9:10. After 4 years ratio will be 11:12. Find Present age of B?

→ $\frac{9n+4}{10n+4} = \frac{11}{12}$

$n = 2$

$B = 10 \times 2 = 20$ yrs

or shortcut

	A		B	
Present →	9	:	10	2 → 4 years
After 4 years →	11	:	12	1 → 2 yrs.

$10 \rightarrow 20$ yrs = B

Q23 Age of A was 5 times of Age of B 5 years ago. and will be three times of B after 3 years. Find Present age of A?

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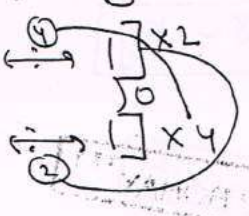
→

	A		B
5 yrs ago →	5	:	1
3 yrs after →	3	:	1



	A		B
	10	:	2
	12	:	4

$2 \rightarrow 8$ yrs

$1 \rightarrow 4$ yrs

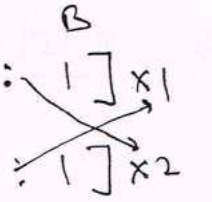
$10 \rightarrow 40$ yrs

Present Age of A = 40 + 5 = 45 yrs

Q24 10 years ago, ratio of ages of A and B was 3:1. After 10 years, ratio of ages will become 2:1. Find the sum of their present ages?

→

	A		B
10 yrs ago →	3	:	1
10 yrs after →	2	:	1



	A		B
	3	:	1
	4	:	2

$1 \rightarrow 20$ yrs

$3 \rightarrow 60$ yrs.

Present age of A = 60 + 10 = 70 yrs

Present age of B = 20 + 10 = 30 yrs

Sum = 70 + 30 = 100 yrs

Q25 Sum of Present ages of A, B and C is 150 yrs. Ratio of their ages 10 yrs ago was 5:4:3. Find ratio of their ages 10 years hence?

→ Sum of Ages 10 yrs ago = $150 - 10 \times 3 = 120$

10 yrs ago → $A = \frac{5}{12} \times 120 = 50$, $B = 40$, $C = 30$

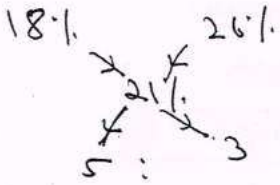
10 yrs hence

A	B	C
70	60	50

Alligation

Q26 A trader has 640 kg of tea, a part of which he sells at 18% Profit & rest at 26% Profit. He gains 21% on whole. What is quantity sold at 18% Profit?

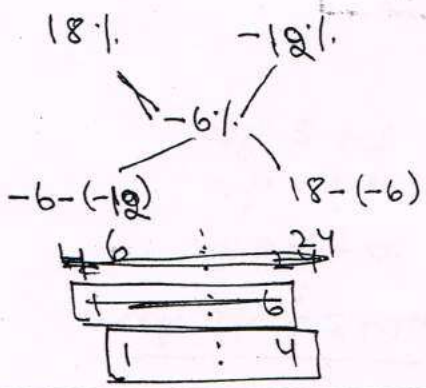
→



$\frac{5}{8} \times \overset{80}{\cancel{640}} = 400$ kg

Q27 A trader has 100 kg of rice, a part of which he sells at 18% Profit and rest at 12% loss. On whole he ~~gains~~ loses 6%. What is quantity sold at 12% loss?

→



$\frac{4}{5} \times \overset{20}{\cancel{100}} =$ **80 kg**

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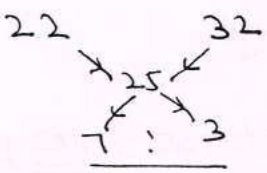
Q28 In what ratio, a grocer mix tea at ₹22/kg and ₹32/kg so that by selling mixture at ₹28/kg, he may gain 12% ?

→ SP = ₹28/kg, η = 12%, CP = $\frac{100}{112} \times 28$ = ₹25/kg



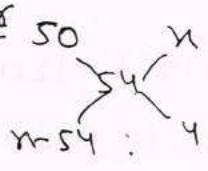
7 : 3 Ans

Q29 A dealer mixes tea costing ₹50/kg with a high quality tea and sells the mixture at ₹54/kg. The ratio in which he mixed the two varieties is 2:1. Find cost of second tea?

→ $\frac{50 \times 2 + x \times 1}{3} = 54$

x = **₹62/kg**

OR

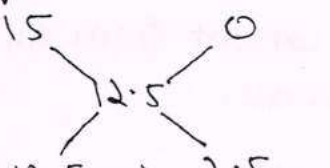


$\frac{x-54}{4} = \frac{2}{1}$

$x = 62$

Q30 In what ratio should water be added to a liquid costing ₹ 15/ltr so as to make a profit of 20% by selling the diluted liquid at ₹ 15/ltr?

→ Liquid water



5 : 1 Ans.

SP = 15, P = 20%

$CP = \frac{100}{120} \times 15 = 12.5$/ltr.

Q31 In what ratio must water be mixed with milk to gain 20% on selling mixture at cost price?

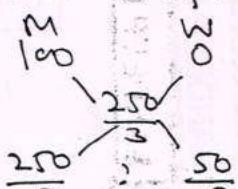
→ **Shortcut**

20% : 100

1 : 5

W M

or Let CP of milk = 100



5 : 1

M : W

SP of mixture = 100

P = 20%

$CP = \frac{100}{120} \times 120 = \frac{250}{3}$

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or 1 : 5

W : M

Q32 A and B are two alloys of Gold and Copper having ratio 7:2 and 7:11 respectively. if equal quantities of alloys are melted to form third alloy C, find ratio of Gold and Copper in alloy C?

→ A: G : Cu = 7 : 2 → $\frac{7}{9}, \frac{2}{9}$

B: G : Cu = 7 : 11 → $\frac{7}{18}, \frac{11}{18}$

C

G : Cu → ?

G → $G_1 + G_2$: Cu → $Cu_1 + Cu_2$

$\frac{7}{9} + \frac{7}{18} : \frac{2}{9} + \frac{11}{18}$

$\frac{21}{18} : \frac{15}{18}$

or

To make both equal

A → 7 : 2 → 9 × (2) → 14 : 4

B → 7 : 11 → 18 × (1) → 7 : 11

21 : 15

Shortcut

7 : 5

Q33 Vessels A and B have milk & water in ratio 4:5 & 5:1. In what ratio should quantities of mixtures be taken from A and B to form a third mixture in which ratio of milk to water is 5:4?

→ A: M:W = 4:5 → $\frac{4}{9}, \frac{5}{9}$; B: M:W = 5:1 → $\frac{5}{6}, \frac{1}{6}$

Take either milk content from all or water content from all.

C: 5:4 → $\frac{5}{9}$ M, $\frac{4}{9}$ W

$\frac{4}{9}$ $\frac{5}{6}$ → $\frac{5}{9}$ → $\frac{5}{18}$: $\frac{1}{9}$ → 5 : 2

OR

$\frac{5}{9}$ $\frac{1}{6}$ → $\frac{4}{9}$ → $\frac{5}{18}$: $\frac{1}{9}$ → 5 : 2

OR

M:W

A → 4:5 → 9 × (2) → 8 : 10

B → 5:1 → 6 × (3) → 15 : 3

C → 5:4 → 9 × (2) → 10 : 8

8 15 → 5 : 2

OR

10 3 → 5 : 2

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Q34 A Container Contains 100 ltrs milk. From this, 10 L of milk was taken out and replaced by water. This process was further repeated two more times. How much milk is now contained by container?

→ Final quantity left of original liquid $= a\left(1-\frac{b}{a}\right)^n$

a → Total (initial) quantity

b → quantity taken out

n → no. of times process done

$100\left(1-\frac{10}{100}\right)^3$

→ $100 \times \frac{729}{1000} =$ 72.9 ltrs

Shortcut

quantity left : Total quantity

36 : 40

$(9)^3 : (10)^3$

729 : 1000

72.9 ltrs : 100 ltr

Q35 8 litres are drawn from a cask full of wine and then replaced by water. This process done three more times. The ratio of quantity of wine now left to that of water is 16:65. How much wine did cask hold originally?

→ $a = 16n + 65n = 81n$

$n = 4$

$b = 8$ ltrs

$\Rightarrow 16n = 81n\left(1 - \frac{8}{81n}\right)^4$

$\sqrt[4]{\frac{16}{81}} = \frac{81n - 8}{81n}$

$\frac{2}{3} = \frac{81n - 8}{81n}$

$162n = 243n - 24$

$n = \frac{24}{81}$

Total Capacity $= 81n = 81 \times \frac{24}{81} = 24$ Ltrs

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Shortcut

Final wine left : Total quantity

16 : 16 + 65

$\sqrt[4]{16} : \sqrt[4]{81}$

2 : 3

1 → 8 Ltrs

3 → 24 ltrs

Q36 A Can contains mixture of liquids A and B in 7:5. When 9 Ltrs mixture taken out and Can is filled with B, ratio of A and B becomes 7:9. Find Initial quantity of mixture and quantity of A in it initially?

→ $\frac{A}{B} \to \frac{7n - \frac{7}{12} \times 9}{5n - \frac{5}{12} \times 9 + 9} \Rightarrow \frac{7n - \frac{21}{4}}{5n - \frac{15}{4} + 9} = \frac{7}{9} \Rightarrow n = 3$

A → $7n = 7 \times 3 = 21$ Ltrs

B → $5n = 5 \times 3 = 15$ Ltrs

Total → 21 + 15 = 36 Ltrs

Shortcut

A		B		
7	:	5	→	12
7	:	9	→	16

same (4)

4 → 9 Ltrs

1 → $\frac{9}{4}$

12 → $\frac{9}{4} \times 12 = 27$ ltrs (−9)

16 → $\frac{9}{4} \times 16 = 36$ Ltr

Ans

→ $A = \frac{7}{12} \times 36^3 = 21$ L

$B = \frac{5}{12} \times 36^3$

15 Ltrs

Q37 A mixture of 150L of milk and water contains 20% water. How many litres of water must be added so that water may be 25% of the mixture?

→

150 L

M 80% 120 L

W 20% 30 L → +10L

120 : 40

$\left[\frac{75}{3} : \frac{25}{1}\right]$

OR

$\frac{M}{W} = \frac{120}{30+x} = \frac{75}{25} = \frac{3}{1}$

$120 = 90 + 3x$

$3x = 30$

$x = 10\,L$

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Q38 Tea worth ₹ 126/kg and ₹ 134/kg are mixed with a third variety in ratio 1 : 1 : 2. if the mixture is worth ₹ 120/kg, find price of third variety?

→

	I	II	III
Quant →	1	1	2
Price	126	134	x

$\frac{126\times1 + 134\times1 + x\times2}{1+1+2} = 120$

$x = 110/kg$

Q39 Three vessels of equal capacity have milk and water in ratio 2:1, 3:1 and 3:2. if all three are emptied into a single large vessel, find ratio of milk and water in final vessel?

→

I	II	III
2 : 1	3 : 1	3 : 2
$\frac{2}{3}, \frac{1}{3}$	$\frac{3}{4}, \frac{1}{4}$	$\frac{3}{5}, \frac{2}{5}$

$\frac{2}{3} + \frac{3}{4} + \frac{3}{5} : \frac{1}{3} + \frac{1}{4} + \frac{2}{5}$

$\frac{40+45+36}{60} : \frac{20+15+24}{60}$

121 : 59

AVERAGE

$$\text{AVERAGE} = \frac{\text{Total sum of all observations}}{\text{Total no. of observations}}$$

or

Total sum = Average × No. of observations.

Q① Average of 11 obs. is 78. if Avg. of first 5 obs. is 72 & Avg of last 7 obs. is 84. Find 5th obs?

→ (1 2 3 4 (5) 6 7 8 9 10 11) Sum of 11 = 11×78 = 858

Sum of 12 [Sum of 1–5 = 5×72 = 360; Sum of 5–11 = 7×84 = 588]

5th obs. = (360+588) − 858 = **90** Ans.

[2] The Average temperature of a week is 34°c. if avg temp. of 1st 3 days is 36°c & Avg temp of last 3 days is 33°c then find the temp of 4th day?

→ (1 2 3) 4 (5 6 7) → 7×34 = 238; 3×36 = 108; 3×33 = 99] 6

4th day = 238 − (108+99) = **31°c** Ans.

---

[3] The avg of 7 numbers is 30. if each number is divided by 5 & then multiplied by 3, then what will be the new Avg.?

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→ New Avg = (30 ÷ 5) × 3 = **18** Ans.

[Same operation done which is done in each obs.]

---

[4] The Avg weight of 19 students is 15 kg. By the admission of a new student the average weight is reduced by 0.2 kg. Find the weight of new student?

→ Total Wt. of 19 → 19×15 = 285 kg

Total Wt of 20 → 20×14.8 = 296 kg

296 − 285 = **11 kg** Ans

Wt of new student

or 20×0.2 = 4 kg (decreased)

so 15 − 4 = 11 kg

[if increased then add]

[5] A batsman score 112 runs in his 24th inning and thus increases his avg by 3.5 runs. Then find his avg which was after 23th inning?

→ Total runs = Avg × No. of Inns.

Let avg after 23 inns = x, Total runs = $23x$

So $23x + 112 = 24(x+3.5)$ → (Total runs after 24 inns.)

$23x + 112 = 24x + 84$ → $x = 28$ Ans.

---

[6] 6 person went to a hotel for taking meals. Five of them spent ₹ 32 each while 6th person spent 80 more than Avg. expenditure of all six. Find total money spent by all?

→ Let avg exp. of all six = ₹ x ⇒ Total exp = $6x$

$5 \times 32 + (x+80) = 6x$ ⇒ $5x = 240$ ⇒ $x = 48$

(6th)

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Total = $6 \times 48 = 288$ Ans.

Or 6th spent 80 more than avg of all six, so divide 80 among rest 5 equally i.e. $\frac{80}{5} = 16$ each

So avg of all six = $32 + 16 = 48$ → Total $6 \times 48 = 288$ Ans.

---

[7] The Avg wt. of 8 people is increased by 3.5 kg when one of them whose wt is 45 kg is replaced by a new man. Find the wt. of new man?

→ Total inc. in wt = 8×3.5 kg = 28 kg.

wt of new man = $45 + 28 = 73$ kg Ans.

---

[8] The Avg. age of 10 people is decreased by 2 years when three men aged 25 yrs, 35 yrs & 40 yrs are replaced by three women. Find Avg age of these three women?

→ Total dec. in age = $10 \times 2 = 20$ yrs.

Total age removed (3 men) = $25 + 35 + 40 = 100$ yrs.

Total age of 3 women = $100 - 20 = 80$ yrs.

Avg age of 3 women = 80 yrs → $26\frac{2}{3}$ yrs Ans.

[9] Mean of 100 items was found to be 30. if at the time of Calculation two items were wrongly taken as 32 & 12 instead of 23 & 11. Find correct mean?

→ Total sum = $100 \times 30 = 3000$

✓	✗
23	32
11	12
34	44

-10

Correct sum = $3000 - 10 = 2990$

Correct mean = $\frac{2990}{100} = \boxed{29.9}$

or change = 10 (to be dec.)

Avg. change = $\frac{10}{100} = 0.1$

Correct mean = $30 - 0.1$

$= \boxed{29.9}$ Ans.

---

[10] 3 yrs ago, Avg. Age of A, B & C was 32 yrs. 4 yrs from now, Avg age of A & C will be 32 yrs. Find Present age of B?

→ Present age of A+B+C = $(32 \times 3) + (3 \times 3) = 105$ yrs.

Present age of A+C = $(32 \times 2) - (4 \times 2) = 56$ yrs.

Age of B = $105 - 56 = \boxed{49 \text{ yrs}}$

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[11] There are 70 students in a class. The Avg marks of the whole class 68. Avg. marks of girls is 80 & that of boys is 60. Find no. of boys in class?

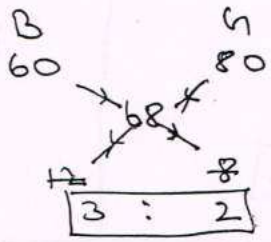
→ Let x boys

then $(70-x)$ girls

$60x + 80(70-x) = 70 \times 68$

$\boxed{x = 42}$

or alligation rule



Boys = $\frac{3}{5} \times 70$

$= \boxed{42}$ Ans.

---

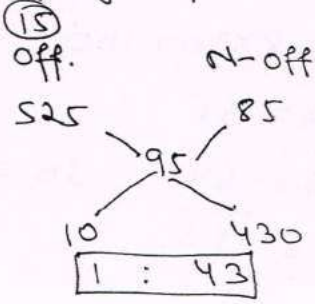
[12] Avg salary per head of all employees is ₹ 95. The Avg salary of 15 officers is ₹ 525 & that of non-officers is ₹ 85. Find total employees?

→ Let total employees = x

$95x = 15 \times 525 + 85 \times (x-15)$

$\boxed{x = 660}$

or alligation



$1 \longrightarrow 15$

$1+43 \longrightarrow 44 \times 15 = \boxed{660}$

13 A Bowler whose bowling Avg is 24, takes 4 wickets for 40 runs in next match & therefore decreases his avg by 1. Find no. of wickets taken by him before last match?

→ Batting Avg = $\frac{\text{Total runs made}}{\text{No. of inns}}$ | Bowling Avg = $\frac{\text{Total runs given}}{\text{No. of wickets taken}}$

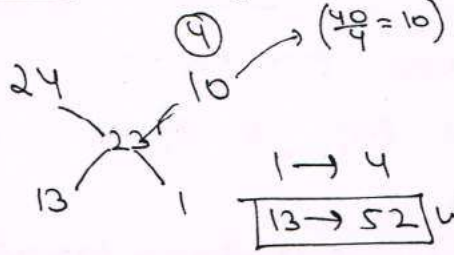
Method 1 → Let wickets taken before last match = x wickets.

Total runs given = $24x$

So $24x + 40 = 23(x+4)$

$x = 52$ wickets

Method 2 → Alligation



14 Out of three numbers, first is twice the second and half of third. If Avg of all three is 56, find third no.?

→ In these type of ques, let any of these nos = x

I	II	III
$2x$	x	$4x$

→ $\frac{2x+x+4x}{3} = 56$

$7x = 168$

$x = 24$

3rd no = $4 \times 24 = 96$

or

I	II	III
$\frac{x}{2}$	$\frac{x}{4}$	x

$\frac{x}{2} + \frac{x}{4} + x = 56 \times 3$

$\frac{2x+x+4x}{4} = 168$

$7x = 168 \times 4$

$x = 96$ = 3rd no.

15 Avg of 8 numbers is 20. Avg of first two is 15.5 & that of next three is $\frac{64}{3}$. The 6th is 4 less than 7th and 7 less than 8th no. Find last no.?

→ Total sum = $8 \times 20 = 160$

$I + II = 15.5 \times 2 = 31$

$III + IV + V = \frac{64}{3} \times 3 = 64$

$I + II + III + IV + V = 95$

6th	7th	8th
x	$x+4$	$x+7$

So $95 + x + x+4 + x+7 = 160$

$3x = 54$

$x = 18$

Last no. = $x+7 = 18+7 = 25$

[16] 3 yrs ago, Avg. age of a family of 5 members was 17 yrs. A baby having born, the avg age is same even today. Find present age of baby?

→ 3 yrs ago, sum of 5 = 17 × 5 = 85 yrs

At Present, sum of 5 = 85 + 5×3 = 100 yrs.

At Present, sum of 6 = 17 × 6 = 102 yr.

Age of baby = 102 − 100 = **2 yrs**

---

[17] The Avg runs of a cricketer in 10 innings was 32. How many runs must he score in his next inning so as to inc. his avg by 4 runs? SANDEEP BOHAY™

→ Total runs in 10 inns = 10×32 = 320

Total runs in 11 inns = 11×36 = 396

11th inning runs = 396 − 320 = **76**

---

[18] A batsman has an avg of 30 runs in 42 inns. The difference between his max. & min. score is 100 runs. if these two innings are removed then Avg reduces to 28. What is his max. score?

→ Total runs in 42 Inns = 42×30 = 1260

Total runs in 40 inns = 40×28 = 1120

Highest = x runs
Lowest = y runs

Difference = 1260 − 1120 = 140 = x + y

& x − y = 100 →

$$\begin{aligned} x+y &= 140 \\ x-y &= 100 \\ \hline 2x &= 240 \end{aligned}$$

→ **x = 120**

---

[19] X has twice money than Y and Y has 50% more money than Z. Avg. money of all three is ₹ 110. money of X?

→ Total money = 110×3 = 330

X	Y	Z
300	150	100
6	3	2

$$X = \frac{6}{11} \times 330 = \boxed{180}$$

20 Avg wt. of A, B & C is 84 kgs. if D joins, Avg wt becomes 80 kg. if E who is 3 kg heavier than D replaces A then avg wt. of B, C, D & E becomes 79 kgs.. wt. of A?

→ $A+B+C = 84\times3 = 252$ kg

$A+B+C+D = 80\times4 = 320$ kg ① → $D = 320-252 = 68$ kg

$E = 68+3 = 71$ kg

$B+C+D+E = 79\times4 = 316$ kg

$B+C+D = 316-71 = 245$ kg — ②

$A = ① - ② = 320-245 = \boxed{75 \text{ kg}}$

---

21 Avg. age of 11 players is increased by 2 months when two of them aged 18 yrs & 20 yrs are replaced by two new players. Find Avg. age of new players?

→ Total inc. in age = $11\times2 = 22$ months = 1 yr + 10 months.

Sum of ages of two new = 18+20+1 yrs + 10 months

= 39 yrs + 10 months

Avg of two new = $\frac{39 \text{ yrs} + 10 \text{ months}}{2}$

= 19.5 yrs + 5 months

= 19 yrs + 6 months + 5 months

= $\boxed{19 \text{ yrs}, 11 \text{ months}}$

PROFIT-LOSS

Profit = SP − CP , Loss = CP − SP

$P\% = \frac{P}{CP} \times 100$, $Loss\% = \frac{L}{CP} \times 100$

$SP = \frac{100+P\%}{100} \times CP$, $\frac{100-L\%}{100} \times CP$

$CP = \frac{100}{100+P\%} \times SP$, $\frac{100}{100-L\%} \times SP$

As Per Law of fraction.
SP > CP if P
SP < CP if L

---

* CP, MP, SP.

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* Let say profit $12\frac{1}{2}\% = \frac{1 \to P}{8 \to CP}$ then SP = 1 + 8 = 9

—— Loss $16\frac{2}{3}\% = \frac{1 \to L}{6 \to CP}$ then SP = 6 − 1 = 5

* Rule of fraction :- if Req. value is greater than Given value then Given value should be multiplied with fraction more than one.

And if Req. value is less than Given value then it should be multiplied with fraction less than one.

- if X% Gain then fraction is $\frac{100+X}{100} \to >1$ or $\frac{100}{100+X} \to <1$
- if Y% Loss then fraction is $\frac{100-Y}{100} \to <1$ or $\frac{100}{100-Y} \to >1$

BASIC-①

① A man bought a cycle for ₹ 560. For how much should he sell it to lose 10%? (504)

[SP < CP so $560 \times \frac{90}{100} = 504$] or [L = 10 → CP, SP = 9; 10 — 560, 9 — $\frac{560 \times 9}{10} = 504$]

② If by selling an article for ₹ 390, a person gains 20%, find his cost price? (325)

[CP < SP so $390 \times \frac{100}{120} = 325$] or [G = 6 → CP, SP = 6 — 390, S — $\frac{390}{6} \times 5 = 325$]

③ A machine is sold for ₹ 5060 at a gain of 10%. What would be the gain% or loss% if it had been sold for ₹ 4370? (5% L)

[$CP = 5060 \times \frac{100}{110} = 4600 > 4370$ (SP), $L\% = \frac{230}{4600} \times 100 = 5\%$]

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④ By selling a watch for ₹ ~~710~~ 1440 a man loses 10%. At what price should he sell it to gain 10%? (1760)

[100 — 90, 110; 90 — 1440, 110 — $\frac{1440}{90} \times 110 = 1760$]

⑤ There would be 10% L if rice is sold at ₹ 5.4/kg. At what price should it be sold to earn a profit of 20%? (7.2/kg)

[100 — 90, 120; 90 — 5.4, 120 — $\frac{5.4}{90} \times 120 = 7.2$]

⑥ By selling an article for ₹ 19.50, a dealer makes a profit of 30%. By how much should he inc. his SP so as to make a profit of 40%? (₹1.5)

[100 — 130 ; 130 — 19.5 ; 140 — $\frac{19.5}{130}$ × 140 = 21 — 19.5 = 1.5]

⑦ A person purchased a mobile at $\frac{9}{10}$th of its SP and sold it at 8% more than its S.P. Find his gain%? (20%)

[Let SP = 100, CP = $\frac{9}{10}$ × 100 = 90 ⟶ SP = 108, G% = $\frac{18}{90}$ × 100 = 20%]

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⑧ Marked price of an article is ₹ 480. Dealer allows a discount of 10% and he ~~gains~~ still gains 8%. if no discount is allowed then find his gain%? (20%)

[SP = 480 × $\frac{90}{100}$ = 432, CP = 432 × $\frac{100}{108}$ = 400, New G% = $\frac{80}{400}$ × 100 = 20%]

⑨ A dealer bought a computer at 20% discount on its original price and sold it at a 40% inc. on original price. find his G%? (75%)

[100 — 80 ; 140 ← ; $\frac{60}{80}$ × 100 = 75%]

⑩ A sold an article to B at 10% L. B sells it back to A at 10% G. who gains and who loses & how much%? (A → −9, B → +10)

[A 100 — 10↓ — B 90 ; 99 — 10↑ — ; A's total Loss = 100 − 90 + 1 = 9% ; $\frac{9}{100}$ × 100 = 9% L ; B's total gain = 99 − 90 = $\frac{9}{90}$ × 100 = 10% G]

⑪ A milkman buys some milk. if he sells it at ₹ 4/ltr, he loses ₹ 100 and if he sells it at ₹ 6/ltr, he gains ₹ 150. How much milk did he purchase? (125 ltrs)

[~~Let 100 ltr~~ ~~6 × 100 − 4 × 100 = 250~~

Let x ltrs
6x − 4x = 150 − (−100) = 250
x = $\frac{250}{2}$ = 125 ltrs

or

by inc. by ₹2/ltr he gets −150 + 100 = 250 as more
so he purchased = $\frac{₹ 250}{₹ 2/\text{ltr}}$ = 125 ltr]

(12) if a discount of 10% is given on MP of an article, the shopkeeper gets a profit of 20%. Find his profit% if he offers a discount of 20% on same article? (20/3%)

[100 — 90 × 100/120 — 75 (MP, SP, CP; 80) P% = 5/75 × 100 = 20/3 %]

(13) what will be percentage profit after selling an article at a certain price if there is a loss of 12½%, when article is sold a half the previous selling price? (75%)

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[CP = 100 — 87.5 = SP/2 → SP = 87.5 × 2 = 175, P% = 75/100 × 100 = 75%]

(14) what will be P% after selling an article at a certain price if there is a loss of 45%, when it is sold at ⅓rd of SP? (65%)

[100 — 55 = SP/3 → SP = 165 P% = 65%]

(15) on basis of SP, the loss is 25%. Find actual L%? (20%)

[SP < CP if SP = 100 then CP → 125, Actual Loss = 25/125 × 100 = 20%]

(16) A merchant find his profit as 20% on SP. Find his actual profit%? (25%)

[SP > CP SP = 100 — CP = 80 Actual P% = 20/80 × 100 = 25%]

(17) if there is a profit of 20% on CP, then %age P on SP is? (16⅔%)

[CP = 100 — SP = 120 20/120 × 100 = 50/3 %]

(18) A man bought two tables for a total cost of ₹900. By selling one for 4/5 of its cost and the other for 5/4 of its cost, he makes a profit of ₹90 on whole. Find cost of lower priced chair? (300)

[4/5 x + 5/4 (900 − x) = 990]

$\pm x \pm y + \frac{(\pm x)(\pm y)}{100}$ = Net P/L — A → B → C - - -

(1) A dealer marks his goods at 20% above CP and then allows a discount of 10%. Find his total P or L%? (+8%).

$\left[20 - 10 - \frac{20 \times 10}{100}\right]$ $\left[100 \underbrace{- 120 - 108}_{+8}\right]$

(2) Find a single discount equal to three successive discounts of 50%, 40% & 20%? (76%)

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(3) A trader allows a discount of 10%. How much % above CP must he mark his goods to make a net profit of 5%? ($16\frac{2}{3}$%)

$\left[x - 10 - \frac{10x}{100} = 5 \rightarrow x = 16\frac{2}{3}\%\right]$ or $\left[\begin{matrix} MP \\ 100 \\ \downarrow \\ 105 \end{matrix} - \begin{matrix} CP \\ 90 \end{matrix} \quad \frac{15}{90} \times 100 = \frac{50}{3}\%\right]$

(4) How much % above CP must a shopkeeper mark his goods so as to earn a profit of 32% after allowing a discount of 12% on MP? (50%)

$\left[\begin{matrix} 100 \\ \downarrow \\ 132 \end{matrix} - 88 \quad \frac{44}{88} \times 100 = 50\%\right]$ or $\left[x - 12 - \frac{12x}{100} = 32\right]$ [

(5) What price should a shopkeeper mark on an article costing him ₹200 to gain 35% after allowing a discount of 25%? (360)

[200 + 35% of 200 = 270 → SP
MP → $270 \times \frac{100}{75} = 360$]

[$x - 25 - \frac{25x}{100} = 35$
$x = 80\%$,
MP = $200 \times \frac{180}{100} = 360$]

[Let MP 100 — SP 75
CP $75 \times \frac{100}{135} = \frac{500}{9}$
% inc. = $\frac{180 - \frac{500}{9}}{\frac{500}{9}} \times 100 = 80\%$,
$200 \times \frac{180}{100} = 360$]

(6) A person marks his goods 25% above CP but allows a discount of 14.5%. if he sells article for ₹875, find his CP? (800)

[CP $\times \frac{125}{100} \times \frac{87.5}{100} = 875$
CP = 800]

⑦ A sells a bicycle to B at a profit of 30%. & B sells it to C at a loss of 20%. : if C pays ₹ 520 for it, at what price did A buy it? (500)

[100 – 130 – 104
104 – 520
100 – 500]

[Rule of fraction (R.o.f)
520 × 80/80 × 100/130 = 500]

⑧ A sells an article to B at 20% P and B sells it to C at 25% P. if C pays ₹ ~~225~~ 450 for it, what was CP for A? (~~150~~) (300)

[R.o.f. ~~225~~ × ~~100~~/125 × 100/120 = 150]

[Initial money = 450 × 100/120 × 100/125 = 300]

⑨ A sells an article to B at 25% P. B sells it to C at 10% P and C sells it to D at 5% P. if C sells it (D buys) ~~for~~ ₹ 231, find CP at which A bought the article? (160)

[R.o.f
I.M. = 231 × 100/105 × 100/110 × 100/125 = 160]

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for Q 7, 8, 9 ⇒

$$\text{Initial value} = \text{Final value} \times \frac{100}{(100 \pm x)} \times \frac{100}{(100 \pm y)} \times \frac{100}{(100 \pm z)} \times \cdots$$

⑩ A shopkeeper bought a table marked at ₹ 200 at successive discounts of 10% & 15%. He spent ₹ 7 on transport and sold table for ₹ 208. Find his gain%? (30%)

[CP → 200 – 23.5% of 200 = 153 + 7 = 160
SP = 208 → P% = 48/160 × 100 = 30%]

Basic-②

① if CP of 10 articles is equal to SP of 9 articles. Find P%.? ($11\frac{1}{9}\%$)

[CP of 10 = SP of 9, $\frac{CP}{SP} = \frac{9}{10}$, P% → $\frac{1}{9} \times 100\%$] [CP = 9, SP = 10, P% = $\frac{1}{9} \times 100\%$]

② if SP of 12 articles is equal to CP of 9 articles. Find L%.? (25%)

[SP of 12 = CP of 9, $\frac{SP}{CP} = \frac{9}{12}$, L% → $\frac{3}{12} \times 100 = 25\%$] [SP = 9, CP = 12, $\frac{3}{12} \times 100 = 25\%$]

③ On selling 25 articles, a shopkeeper gains by SP of 5 articles. Find G%.? (25%)

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[Let SP of 1 = 1, SP of 25 = 25, CP = 25 − 5 = ₹20, G% = $\frac{5}{20} \times 100 = 25\%$] [if loss (or losses) then CP = 30, L% = $\frac{5}{30} \times 100 = \frac{50}{3}\%$]

④ A person sold 16 articles for the same money as he paid for 20. What is his gain%.? (25%)

[SP of 16 = CP of 20, $\frac{SP}{CP} = \frac{20}{16}$, G% = $\frac{4}{16} \times 100 = 25\%$]

⑤ A wholeseller sells 30 pens for the price of 27 pens to a retailer. The retailer sells the pens at marked price. Find %P for retailer? ($11\frac{1}{9}\%$)

[Let MP of 1 = 1, MP of 30 = 30, MP of 27 = 27, CP for retailer = 27, SP ——— = 30, %P = $\frac{3}{27} \times 100 = 11\frac{1}{9}\%$]

⑥ By selling 66 mtrs of cloth, a person gains the cost of 22 mtrs. Find his gain%.? ($33\frac{1}{3}\%$)

[G% = $\frac{22}{66} \times 100 = \frac{100}{3}\%$] → P = SP − CP

CP of 22 = SP of 66 − CP of 66

CP of 88 = SP of 66

$\frac{CP}{SP} = \frac{66}{88} = \frac{6}{8} = \frac{3}{4}$ so P% = $\frac{1}{3} \times 100\%$

IF IT HAD BEEN ---

① A person sold a chair at 12% P. Had he sold it ~~for~~ at a gain of 18%, he would have gained ₹24 more? Find his cost price? (400)

CP: 100 — 112, 118 / 6 → 24, 100 → 400

CP : SP
100x : 112x
$\frac{100x}{112x+24} = \frac{100}{118} \rightarrow x = 4$, CP = 400

$\frac{118}{100}x - \frac{112}{100}x = 24$

② A person sold an article at a loss of 10%. if he had bought it for 20% less and sold it for ₹55 more, he would have had a profit of 40%. Find CP of article? (250)

CP: 100 — 90, 80 — 112 / 22 → 55, 100 → 250

CP : SP
100 : 90
10x : 9x
$\frac{10x}{9x} \Rightarrow \frac{8x}{9x+55} = \frac{100}{140} \rightarrow 56x = 45x + 275$
$x = 25$, CP = 10 × 25 = 250

③ A man sells an article at a profit of 20%. if he had bought it at 20% less and sold for ₹75 less, he would have gained 25%. CP? (375)

CP: 100 — 120, 80 — 125/100 × 80 = 100 / 20 → 75, 100 → 375

$\frac{10x}{12x} \Rightarrow \frac{8x}{12x-75} = \frac{100}{125} \rightarrow x = 37.5$
CP = 375

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④ A dealer sold a radio at a loss of 2.5%. Had he sold it for ₹100 more, he would have gained 7.5%. For what value should he sell it to gain 12.5%? (1125)

100 — 97.5 / 107.5 — 10 → 100, 1 → 10 / 112.5 → 1125

⑤ An article is sold at 20% P. if both CP and SP were ₹100 less, the profit would be 4% more. Find CP? (600)

CP : SP
100 : 120
10x : 12x
$\frac{10x-100}{12x-100} = \frac{100}{124} \rightarrow 1240x - 12400 = 1200x - 10000$
$40x = 2400$
$x = \frac{2400}{40} = 60$, CP = 60 × 10 = 600

⑥ An article is sold at 10% P. Had it been bought for ₹10 less and sold for ₹20 more, then profit = 20%. Find CP? (320)

$\frac{10x-10}{11x+20} = \frac{100}{120} \rightarrow x = 32$, (CP = 32 × 10 = 320)

⑦ A fruit merchant makes a profit of 25% by selling oranges at a certain price. If he charges ₹1 more on each mango, he would gain 50%. Find what price per mango did he sell at first? (₹5)

100 — 125
25 — 1
150 — 125 — 25

$\frac{100x}{125x} \Rightarrow \frac{4x}{5x+1} = \frac{2}{3} \Rightarrow 12x = 10x+2$

$x = 1$; $SP_1 = 5 \times 1 = 5$

Q→ A vendor sells some toffees at 20 for a rupee gaining 40%. How many ~~must he~~ did he buy for a rupee? (28)

→ CP : SP
100 : 140
→ 10 : 14 [Lesser articles, more profit]

No. of articles $\propto \frac{1}{\text{value}}$

No. → 14 : 10 (Inversely proportional)
×2 ↓ ↓ ×2
(28) 20

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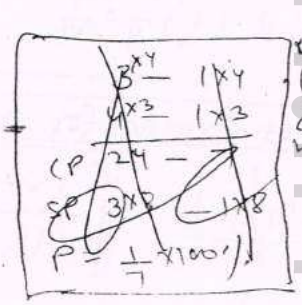
Q→ A man purchases certain no. of ~~op~~ articles at 3 for a rupee and same no. of articles at 4 for a rupee. He mixes them and sell at 3 for a rupee. Find Gain%/Loss%?

→

	No.	₹
CP_1	3×4 —	1×4
CP_2	4×3 —	1×3
Mix CP	$24^{\times 1}$ —	7×1
SP	$3^{\times 8}$ —	$1^{\times 8}$

Mix CP ⇒ 24 → 7
SP ⇒ 24 → 8 } +1

Gain% = $\frac{1}{7} \times 100 = \frac{100}{7}\%$ G



(75)

for a rupee

① A man purchases 11 toffees for ₹10 and sells 10 toffees for ₹11. Find his %P or %L? (+21%)

CP of 1 = 10/11, SP of 1 = 11/10, %P = (11/10 − 10/11)/(10/11) × 100 = 21%

or

Purchases 11 for 10 ₹, sells 10 for 11 ₹, (11×11 − 10×10)/(10×10) × 100 = +21% = P

② A boy buys apples at 9 for ₹16 and sells them at 11 for ₹20. Find his %P or %L? (+25/11 %)

P 9 — 16, S 11 — 20, (9×20 − 11×16)/(11×16) × 100 = 25/11 %P

or

CP 9×11 — 16×11 → 176, SP 11×9 — 20×9 → 180, 4/176 × 100

③ A vendor sells some articles at 10 for a rupee gaining 40%. How many toffees did he buy for a rupee? (14)

SP of 1 = 1/10, so (14); CP of 1 = 100/140 × 1/10 = 1/14

x — 1, 10 — 1, (x − 10)/10 × 100 = 40 → x = 14

CP SP, 100 → 140, 10 → 14, 40 → 4, 10

= (14) (10)

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④ If some articles are bought at the rate 30 for a rupee, how many must be sold for a rupee to gain 25%? (24)

CP of 1 = 1/30, so (24); SP of 1 = 125/100 × 1/30 = 1/24

30 — 1, x — 1, (30 − x)/x × 100 = 25, 120 − 4x = x → x = 24

30×x — 1×x, x×30 — 1×30, (30 − x)/x × 100 = 25

⑤ A man purchases certain no. of articles at 3 for a rupee and same no. of articles at 4 for a rupee. He mixes them and sell at 3 for a rupee. Find his G% or L%? (+100/7 %)

CP of 1st = 1/3, CP of 2nd = 1/4, CP of 2 = 1/3 + 1/4 = 7/12

CP of mix (1) = 7/24, SP of 1 = 1/3

G% = (1/3 − 7/24)/(7/24) × 100 = 100/7 %

or

Let he bought 12

(2×3×4/(3(3+4)) − 1) × 100, 3/21 × 100 = 100/7 %

Direct formula for this type.

%G/%L → [2xy/(z(x+y)) − 1] × 100

+ → P

− → L

Buys x for a rupee

— y for a rupee

&

sells z for a rupee

→ %

(42)

(6) Some Articles are bought at 11 for a rupee and an equal no more at 9 for a rupee. if these are sold at 20 for 2 rupees, find Loss % or G%? (−1%)

$\left(\frac{2xy}{2(x+y)} - 1\right) \times 100 \rightarrow \left(\frac{2\times11\times9}{10(11+9)} - 1\right) \times 100$ for a rupee

$\Rightarrow \left(\frac{198}{200} - 1\right) \times 100 \rightarrow \frac{-2}{200} \times 100 = -1\%\ L$

He bought 99 for 9 rs and 99 for 11 rs → 198 for 20 rs

Sold 198 for 19.8 rs → Loss $= \frac{0.2}{20} \times 100 = -1\%\ L$

(7) A person bought some oranges at ₹30 a dozen and an equal no at ₹40 a dozen. He sold them at ₹45 a dozen and made a profit of ₹480. Find the no. of dozen of oranges he bought? (48)

Let he bought 100 dozen for ₹3000 & sold 200 dozen for ₹ 45×200 = 9000

100 — ₹4000

Total 200 — 7000

P = 9000 − 7000 = 2000

Now 2000 — 480

1 — $\frac{480}{2000}$

200 — 48

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(8) By selling pencils at 15 a rupee, a person loses 9%. How many for a rupee must he sell to gain 5%? (13)

$\frac{1}{15} \times \frac{20}{91} = \frac{20}{3\times91} \rightarrow$ (P of 1)

now $\frac{20}{3\times91} \times \frac{105}{100} = \frac{1}{13}$ so 13

or

(CP 100 / 105) 91 — 91 − 15 = 15 — 105 − 105 = 91×15 — $= \frac{1}{13} \rightarrow (13)$

or 105 − 15 = 1 − 15/105 = 91 − $\frac{15}{105}\times91$ = (13)

(For Profit, lesser pencil should be sold)

(9) Some toffees were bought at 11 for ₹10 and equal no at 9 for ₹10. Whole lot was sold at one rupee per toffee. P% or L%? (1% L)

11×9 — 10×9

9×11 — 10×11

CP → 198 — 200

SP of 198 — ₹198

$L = \frac{2}{200} \times 100 = 1\%\ L$

CP	SP1	SP2
100	91	105
	13	15
No →	15 :	13
	(15)	(13)

DISHONEST DEALER

$$\%\ G \text{ on wt.} = \frac{\text{Diff. in wts}}{\text{false wt.}} \times 100$$

① A dishonest dealer sells his goods at CP but uses a wt of 920 gm instead of a kg wt. Find his gain%? $\left[\frac{80}{920}\times 100 = \frac{200}{23}\%\right]$

② A grocer sells rice at 10% P and also uses a wt which is 20% less. Find his total %G? (37.5%)

[%G on wt. $= \frac{20}{80}\times 100 = 25\%$

Total $\rightarrow 25 + 10 + \frac{250}{100} = 37.5\%$]

[20% less wt means $= \frac{20}{80}$

10% P on 100 $\rightarrow$ +10 gm

Total G% $= \frac{20+10}{80}\times 100 = 37.5\%$]

[100 G 110, 80, $\frac{30}{80}\times 100$]

③ A seller uses 840 gm wt instead of 1 kg wt and also sells his goods at 4% gain on CP. Find his actual % profit? $\left(\frac{500}{21}\%\right)$

[%G on wt. $= \frac{160}{840}\times 100 = \frac{400}{21}\%$

Total G $\rightarrow \frac{400}{21} + 4 + \frac{1600}{2100} = \frac{500}{21}\%$]

[4% G on 1000 gm = 40 g

Total G% $= \frac{160+40}{840}\times 100 = \frac{500}{21}\%$]

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④ A dealer sells his goods at 20% loss on CP but uses 40% less wt. What is his net P% or L%? $(33\frac{1}{3}\%\ P)$

[$\frac{40}{60}\times 100 = \frac{200}{3}\%$

Net $= \frac{200}{3} - 20 - \frac{200\times 20}{3\times 100} = \frac{100}{3}\%$ +]

[20% L on 100 = −20 gm

Total $= \frac{40+20}{60}\times 100 = \frac{100}{3}\%$]

[100 L 80, 4, 60, $\frac{20}{60}\times 100 = \frac{100}{3}\%$]

⑤ A dishonest shopkeeper sells at 10% P on CP and also uses a false wt of 800 gm for 1 kg. Find his total P%? (37.5%)

[$\frac{200}{800}\times 100 = 25\%$

$25 + 10 + \frac{250}{100} = 37.5\%$]

[10% P on 100 = +100 gm

Total $\rightarrow \frac{200+100}{800}\times 100 = 37.5\%$]

[100, 110, $\frac{35}{75}\times 100$, 75]

⑥ A dishonest dealer sells his goods at $6\frac{1}{4}$% loss on CP but uses 14 gm wt. instead 16 gm. Find his actual %P or %L? ($50/7$ %+)

$\frac{2}{14} \times 100 = \frac{100}{7}$%.

$\frac{100}{7} - \frac{25}{4} - \frac{2500}{2800} = \frac{50}{7}$%.

$\frac{25}{4}$%. L on 16 gm $= \frac{25}{4 \times 100} \times 16 = 1$ gm

Total $= \frac{2-1}{14} \times 100 = \frac{50}{7}$%.

⑦ A dishonest dealer sells at CP but uses a false wt and gains 25%. Find false wt used? (Instead of 1 kg) (800 gm)

$\frac{1000 - x}{x} \times \frac{4}{100} = \frac{25}{100}$

$4000 - 4x = x$

$x = \frac{4000}{5} = 800$ gm

25% $= \frac{1}{4} \to P$ $\quad SP = 4+1 = 5$

$5 \to 1000$ gm

$4 \to 800$ gm

CP — SP

100 — 100

$\frac{100}{125} \times 100 = 80$

false wt.

800 gm for 1 kg.

⑧ A dishonest dealer defrauds by using false balance to the extent of 10% in buying as well as selling goods. Find his gain%?

100 — 110 (CP — SP)

$x\% = \frac{20}{90} \times 100 = \frac{200}{9}$%.

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Two or more parts

① if some articles are purchased for ₹450 and $\frac{1}{3}$rd be sold at a loss of 10%. At what gain% should the rem. be sold so as to gain 20% on whole transaction? (35%)

$\frac{1}{3}$rd + $\frac{2}{3}$rd = 1

−10 , x

+20

$20 \times 1 = \frac{1}{3}(-10) + x \times \frac{2}{3}$

$x = 35\%$

$\frac{1}{3} : \frac{2}{3}$

−10% , x%

20%

x−20 : 20−(−10)

1 : 2

$\frac{x-20}{30} = \frac{1}{2}$

$x = 35\%$

② $\frac{2}{3}$rd of a consignment was sold at a profit of 6% and the rest at a loss of 3%. if there was an overall profit of ₹540, find the value of consignment? (18000)

$\frac{2}{3} + \frac{1}{3} = 1$

6 , −3 , x

$x \times 1 = \frac{2}{3} \times 6 - \frac{1}{3} \times 3$

$x = 3\% \text{ — } 540$

$100\% \text{ — } 18{,}000$

2 : 1

6 , −3 , x

x−(−3) , 6−x

$\frac{x+3}{6-x} = \frac{2}{1} \rightarrow x = 3\%$

$3\% \text{ — } 540$

$100\% \text{ — } 18000$

③ $\frac{1}{3}$rd of a commodity is sold at 15% P, $\frac{1}{4}$ is sold at 20% P and the rest at 24% P if a total profit of ₹62 is made, then find the value of commodity? (310)

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$\frac{1}{3} + \frac{1}{4} + \frac{5}{12} = 1$

15% , 20% , 24% , x%

$x \times 1 = \frac{1}{3} \times 15 + \frac{1}{4} \times 20 + \frac{5}{12} \times 24$

$x = 20\%$ so $20\% \text{ — } 62$

$100\% \text{ — } 310$

④ A man purchased 20 dozen notebooks at ₹48/dozen. He sold 8 dozens at 10% P and rem. 12 dozen at 20% P. What is his total P%? (16%)

8 + 12 = 20

10% , 20% , x

$20 \times x = 80 + 240$

$x = \frac{320}{20} = 16\%$

8 , 12

10 , 20

x

20−x , x−10

$\frac{20-x}{x-10} = \frac{8}{12} = \frac{2}{3}$

$x = 16\%$

(5) A dealer purchased medicines of ₹6000. He sold $\frac{1}{3}$ Part at 30% L. At what gain% should he sell rest so that he has neither gain nor loss? (15%)

$\frac{1}{3}\times(-30)+\frac{2}{3}\times x=0$

$x=15\%$.

1 : 2

−30 , x → 0 ; x , 0−(−30)

$\frac{x}{30}=\frac{1}{2}$

$x=15\%$.

(6) A dealer buys 160 kg rice @ 27/kg and 240 kg rice @ 32/kg. At what rate/kg should he sell the mixture to gain 20%? (36)

$\frac{160\times27+240\times32}{400}$

$=30\times\frac{120}{100}=36/\text{kg}$

2 : 3

27 , 32 → x ; 32−x , x−27

$x=30\rightarrow 30\times\frac{120}{100}=36/\text{kg}$.

$\frac{2\times27+3\times32}{5}=30$

$30\times\frac{120}{100}=36/\text{kg}$

(7) A man purchases 5 tables and 10 chairs for ₹10,000. He sells the tables at 15% profit and chairs at 10% loss. and thus he gets ₹375 as profit. Find cost of 1 table & 1 chair? (1100, 450)

Let CP of 1 table = x

$\frac{15}{100}\times5x-\frac{10}{100}(10{,}000-5x)=375$

$x=1100$, chair $\rightarrow\frac{10000-5500}{10}=450$

T , C → 15 , −10 ; 375 ; 13.75 , 11.25

11 : 9.

CP of 5 tables $=\frac{11}{20}\times10000=5500$

1 table $=\frac{5500}{5}=1100$

CP of 10 chairs $=\frac{9}{20}\times\frac{10000}{10}=450$

(8) A man buys two articles for ₹1350. He sells one at a loss of 6% and other at a gain of 7.5%. on the whole, he neither gains nor loses. what does each article costs? (750, 600)

$-6x+7.5y=0$

$\frac{x}{y}=\frac{75}{60}=\frac{5}{4}$ so $x=\frac{5}{9}\times1350=750$, $y=600$

−6 , 7.5 → 0 ; −7.5 , 6 ; 5 : 4

$\frac{5}{9}\times1350=750$

$\frac{4}{9}\times1350=600$

$x\times\frac{94}{100}+(1350-x)\times\frac{107.5}{100}=1350$

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⑧ A person bought two watches for ₹480. He sold one at a loss of 15% and other at a gain of 19% and he found that each watch was sold at same price (SP same). Find CP of both? (280, 200)

$x \times \frac{85}{100} = (480-x)\left(\frac{119}{100}\right)$

$x = 280$, $480-280 = 200$

$x_1 \times \frac{85}{100} = x_2 \times \frac{119}{100}$

$\frac{x_1}{x_2} = \frac{7}{5} \rightarrow x_1 = \frac{7}{12} \times 480 = 280$, $x_2 = 200$

Direct (for same SP)

$$\text{CP of Loss} = \frac{480 \times (100+P\%)}{(100+P\%)+(100-L\%)} = \frac{480 \times 119}{119+85} = 280$$

$$\text{CP of Gain} = \frac{480 \times (100-L\%)}{(100+P\%)+(100+L\%)} = \frac{480 \times 85}{119+85} = 200$$

⑨⑩ A person sold his watch for ₹144 and his % age of profit is equal to CP. Find CP of watch? (80) (hst)

$x = \frac{144 \times 100}{(100+x)} \rightarrow x^2 + 100x - 14400 = 0$

$x = -180, (80)$

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~~⑪ Each of two cars is sold at~~

⑪. Each of the two articles is sold for ₹720. The 1st is sold at 25% P and other at 25% L. What is % Loss or Gain in total? (−6.25%)

$\frac{x^2}{100} = -6.25\%\ L$ ~~Total Loss = $\frac{6.25}{100} \times 1440 = \frac{144 \times 10000}{1000 \times 16}$~~ = ~~or if SP = 100 − 6.25~~

$CP = 1440 \times \frac{100}{(93.75)} = 1536$, $\text{Loss} = 1536 - 1440 = 96$

⑫ A man sells two articles for ₹5000 each neither losing nor gaining in the deal. If he sold one at a ~~loss of~~ gain of 25% ~~on~~ then other is sold at a loss of ? $(-\frac{50}{3}\%)$

CP of 1st = $5000 \times \frac{100}{125} = 4000$

CP of 2nd = $10000 - 4000 = 6000$

L% = $\frac{1000}{6000} \times 100 = \frac{50}{3}\%$

4 : 25 — 6 : $-x$

x : 25

$\frac{x}{25} = \frac{4}{6} \rightarrow x = \frac{50}{3}\%$

Miscellaneous

① CP of 3 balls = CP of 2 Pads
CP of 3 Pads = CP of 2 gloves
CP of 3 gloves = CP of 2 bats
and if CP of 1 bat = ₹ 54
then CP of 1 ball ₹ ? (16)

$$\text{CP of 1 ball} = \frac{54 \times 2 \times 2 \times 2}{3 \times 3 \times 3} = 16$$

② CP of 9 kg rice = CP of 4 kg sugar
—— 14 kg sugar = —— 1.5 kg tea
—— 2 kg tea = CP of 5 kg Coffee
and if cost of 11 kg coffee = ₹ 462
then find cost of 2.5 kg rice? (₹ 12.5)

$$\text{CP of 1 kg rice} = \frac{462 \times 5 \times 1.5 \times 4}{9 \times 14 \times 2 \times 11} = ₹ 5$$

CP of 2.5 kg rice = $2.5 \times 5 = 12.5$

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③ A man sells two horses for ₹ 1710. CP of 1st is equal to SP of second. if 1st is sold at 10% loss and second at 25% gain, what is his total gain or loss? (₹ 90)

Let CP of 1st = 100
SP of 1st = 90
SP of 2nd = 100
CP of 2nd = $100 \times \frac{100}{125} = 80$

Total CP = 100 + 80 = 180
Total SP = 100 + 90 = 190

For SP 190 gain is —— ₹ 10
—— 1710 —— —— $\frac{10}{190} \times 1710 = 90$ ₹

$\% = \frac{10}{180} \times 100 = \frac{50}{9}\%$

TIME & WORK

* $M_1\ W_1\ D_1\ T_1$
  $M_2\ W_2\ D_2\ T_2$
  $[M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1]$

* Work ∝ Person (men)

  Time ∝ $\frac{1}{\text{Men}}$

  Work ∝ Time

  if work is same then

  $[M_1 D_1 = M_2 D_2]$

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* A can do a work in 10 days, B → 12 days, C → 15 days. How long will they take to do it together?

  units/day or efficiency

→ A → 10 days
  B → 12 days
  C → 15 days

Total units = 60 (L.C.M.)

A → 6
B → 5
C → 4

A + B + C = 15 (efficiency)

Days = $\frac{60}{15}$ = 4 days

---

* ~~A alone takes~~

Q① 16 M can do a work in 10 days. Find no. of men to complete work in 20 days? $[M_1D_1 = M_2D_2]$ (8)

② 40 men can cut 60 trees in 8 hrs. if 8 men leave the job, how many trees will be cut in 12 hrs? (72)

$$\left[\begin{array}{l} 40 - 60 - 8 \\ 32 - x - 12 \end{array} \rightarrow x = 72\right]$$

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③ 5 men can make 10 toys in 6 days working 6 hrs a day. In how many days can 12 men make 16 toys working 8 hrs a day? (3 D)

$$\left[\begin{array}{l} 5 - 10 - 6 - 6 \\ 12 - 16 - x - 8 \end{array} \rightarrow x = 3\right]$$

④ A certain no. of men can do a work in 60 days. if there were 8 men more, it could be finished in 10 days less. Find original no of men?

$$\left[\begin{array}{l} x - 1 - 60 \\ x+8 - 1 - 50 \end{array} \rightarrow x = 40\right] \text{ or } [M_1D_1 = M_2D_2] \quad (40)$$

⑤ A group of men decided to do a work in 10 days, but five of them became absent. if rest of the group did work in 12 days, find original no. of men?

$$\left[\begin{array}{l} x - 1 - 10 \\ x-5 - 1 - 12 \end{array} \rightarrow x = 30\right] \quad [M_1D_1 = M_2D_2] \quad (30)$$

⑥ There is sufficient food for 400 men for 31 days. After 28 days, 280 men leave the place. for how many days will the rem. food last for rest of the men? (10 D)

if no one leaves then → 3 days
so for 400 → 3 days
— 1 → 1200 days
— 120 → $\frac{1200}{120} = 10$ days

or $\left[\text{Days} = \frac{\text{Rem. food}}{\text{Rem. men}} = \frac{400(31-28)}{400-280} = 10 \text{ D}\right]$

⑦ A builder decided to build a house in 40 days. He employed 100 men in start & 100 more after 35 days. & completed work in time. if he had not employed additional men, how many days behind schedule would it have been finished? (5 D)

200 M do rest work in — 5 D
100 M —— ——— → 10 D

So 35 + 10 = 45 D. Total time
45 − 40 = 5 Days behind schedule

⑧ A contractor undertakes to dig a canal 12 km long in 350 days. and employs 45 men. After 200 days, he finds that only 4.5 km canal has been completed. Find no. of extra men needed to finish the work in time? (55)

45 — 4.5 — 200
x — 7.5 — 150

→ x = 100 men, additional → 100 − 45 = 55

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⑨ 1 Man or 2 women or 3 Boys can do a work in 44 days. Then in how many days will 1 Man, 1 woman and 1 Boy do that work together? (24 days)

→ 1 M = 2 W = 3 B

1 M = 3 B
1 W = $\frac{3}{2}$ B

So 1 M + 1 W + 1 B
↓
3 B + $\frac{3}{2}$ B + 1 B = $\frac{11}{2}$ B

$M_1D_1 = M_2D_2$

$3 \times 44 = \frac{11}{2} \times x$

x = 24 days

(87) **BASIC-2**

① If 3 Men or 4 women can reap a field in 43 days, how long will 7 men and 5 women take to reap it? (12 D)

$3M = 4W$

$1M = \frac{4}{3}W$

$7M + 5W \rightarrow \frac{28}{3} + 5 = \frac{43}{3}W$

$4 - 1 - 43$

$\frac{43}{3} - 1 - n$ $\quad n = 12\,D$

$M_1D_1 = M_2D_2$

OR

$\text{Days} = \left[\frac{1}{\frac{7}{43\times3} + \frac{5}{43\times4}}\right]$

$= \frac{43\times3\times4}{7\times4+3\times5} = 12\,D.$

② 10M or 12W can do a work in 10 days. How many days will 15 M and 6W take to do that work? (5 D)

$$\frac{1}{\frac{15}{10\times10} + \frac{6}{12\times10}} = \frac{1}{\frac{3}{20} + \frac{1}{20}} = \frac{20}{4} = 5\ \text{Days}$$

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③ 8C + 12M = 9 days. Each child takes twice the time taken by a man. Time taken by 12 M? (12 D)

$1M = 2C \rightarrow 8C + 12M = 4M + 12M = 16M$ so

$M_1D_1 = M_2D_2$

$16 - 1 - 9$

$12 - 1 - n$ $\quad n = 12$

④ $4M + 6W = 8\,D$, $3M + 7W = 10\,D$

Time taken by 10W? (40 D)

$8(4M+6W) = 10(3M+7W)$

$1M = 11W$ so $4\times11W + 6W = 8D \rightarrow 50W$ takes 8 days

$10W \text{ —— } 8\times5 = 40D$

$M_1D_1 = M_2D_2$

⑤ $24M + 32B = 5\,D$, $26M + 48B = 4\,D$, $14M + 20B = ?$ $\left(\frac{25}{3}\,D\right)$

$5(24M+32B) = 4(26M+48B) \rightarrow 1M = 2B$

$M_1D_1 = M_2D_2$

so $24\times2B + 32B = 80B$

$14\times2B + 20B = 48B$ $\rightarrow$

$80 - 1 - 5$

$48 - 1 - n$ $\quad n = \frac{25}{3}$ Days

A + B + C −

① A = 5 Days, B = 6 days, A + B = ? ($\frac{30}{11}$ Days)

[5 / 6 → T.U. 30 → U.d. 6, 5 ; $\frac{30}{11}$ days]

[$\frac{1}{6} + \frac{1}{5} \rightarrow \frac{11}{30} \rightarrow \frac{30}{11}$]

② Mohan = 10 D, Ramesh = 15 D, Suresh = 30 D, M + R + S = ? (5 D)

[10 / 15 / 30 → T.U. 30 → U/d → units/day 3, 2, 1 ; $\frac{30}{3+2+1} = 5$ D]

[$\frac{1}{10} + \frac{1}{15} + \frac{1}{20} \Rightarrow \frac{6}{30} \rightarrow \frac{30}{6} = 5$]

③ A + B = 8 days, B + C → 6 days, C + A = 10 days. A + B + C = ? ($\frac{240}{47}$ D)

[A+B — 8 / B+C — 6 / C+A — 10 → 120 → 15, 20, 12

$2(A+B+C) = 15+20+12 = 47$

$A+B+C = \frac{47}{2}$, Days $= \frac{120 \times 2}{47} = \frac{240}{47}$]

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④ A + B = 12 Days, A = 18, B alone = ? (36 D)

[A+B — 12 / A — 18 → 36 → 3, 2 ; B = 3 − 2 = 1 ⇒ $\frac{36}{1}$ = 36 Days]

⑤ A + B + C = 12 days, A → 36 days, B → 54 days, C = ? (27 D)

[A+B+C — 12 / A — 36 / B — 54 → 108 → 9, 3, 2

C = A+B+C − A − B = 9 − 3 − 2 = 4 ; D = $\frac{108}{4}$ = 27]

⑥ A + B = 12, B + C = 15, C + A = 20. A = ?, B = ?, C = ? (30, 20, 60)

[A+B — 12 / B+C — 15 / C+A — 20 → 60 → 5, 4, 3

$A+B+C = \frac{5+4+3}{2} = 6$ units/day

$A = 6 - 4 = 2 \rightarrow D = \frac{60}{2} = 30$ D

$B = 6 - 3 = 3 \rightarrow \frac{60}{3} = 20$ D

$C = 6 - 5 = 1 \rightarrow \frac{60}{1} = 60$ D]

(7) A+B+C = 6 days, A+B = 8 days, B+C = 12 days. C+A = ? (8 D)

A+B+C — 6
A+B — 8 → 24 → 4, 3, 2
B+C — 12

A = 4 − 2 = 2 u/d
C = 4 − 3 = 1 u/d
A+C = 2+1 = 3 u/d, D = $\frac{24}{3}$ = 8 days

(8) 5 M can do a work in 4 days, 10 W in 6 days and 5 B in 8 days. 1 M + 1 W + 1 B = ? ($\frac{120}{11}$ Days)

20 M — 1 D
60 W — 1 D
40 B — 1 D
$\frac{1}{20} + \frac{1}{60} + \frac{1}{40} = \frac{11}{120} \rightarrow \frac{120}{11}$ Days

or
1 M — 20 D
1 W — 60 D → 120 → 6, 2, 3
1 B — 40 D

$\frac{120}{11}$ Days

(9) Nayan → 5 days working 4 hrs a day
Ravi → 10 days working 5 hrs a day.
Nayan + Ravi together working 10 hrs a day? ($\frac{10}{7}$ D)

Nayan 1 hr a day → 20 D, 10 hrs a day → 2 D
Ravi 1 hr a day → 50 D, 10 hrs a day → 5 D
N — 2
R — 5
→ 10 → 5, 2

$\frac{10}{7}$ Days

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(10) A alone takes 27 days more than A and B together take.
B alone takes 3 days more than A and B together take.
then find time taken by A & B together to do work?

→ Shortcut → A+B → $\sqrt{27 \times 3}$ = 9 days.

or Let together A & B → n days
A → n + 27
B → n + 3
so $\frac{1}{n+27} + \frac{1}{n+3} = \frac{1}{n}$

ALTERNATIVELY

① A → 10 days, B → 15 days
They work on alternate days & A starts 1st. (12 D)
Find total time taken to complete work?

A–10, B–15 → 30 → 3, 2
A B A B
3 2 3 2
In 2 days – 5 units
30 units in → 2×6 = 12 days

② A → 12 days, B → 15 days, C → 20 days. (7 D)
A+B A+C A+B A+C ------ Total days?

A–12, B–15, C–20 → 60 → 5, 4, 3
A+B 9 | A+C 8 | A+B 9 | A+C 8 | A+B 9 | A+C 8 | A+B 9
17 + 17 + 17 + 9 = 60 units
2 + 2 + 2 + 1 = 7 days

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③ A → 8 hrs, B → 12 hrs. Work alternately for 1 hr & A starts at 10 a.m. Time to finish the work? (7:30 P.M.)

A–8, B–12 → 24 → 3, 2
A B A B --
3 2 3 2 ---
5
5×4 = 20 in 4×2 = 8 hrs.
Next 3 units in 1 hr by A
Last 1 unit in ½ hr by B
Total time = 8 + 1 + ½ = 9½ hrs → 10:00 + 9½ hrs = 7:30 PM

④ A → 9 days, B → 12 days. Work alternatively and A starts. Total time to finish the work? (10¼ Days)

A–9, B–12 → 36 → 4, 3
A B A B
4 3 4 3
7
7 units in 2 days
7×5 = 35 units — 2×5 = 10 days
Rem 1 unit by A in ¼ day so 10¼ days

EFFICIENCY

① $A = 2B$, $A+B = 15$ days. A alone, B alone? (22.5, 45 D)

$B \to 1$ unit/d
$A \to 2$ unit/d
$A+B = 3$ unit/day
Total units $= 15 \times 3 = 45$ units
$A = \frac{45}{2} = 22.5$ D
$B = \frac{45}{1} = 45$ D

or

$A = x$ days
$B = 2x$ days
$\frac{1}{x} + \frac{1}{2x} = \frac{1}{15}$
$x \to 22.5$

② A = 200% more efficient than B and takes 60 days less than B. A+B = ? (22.5 D)

	Eff	Days
A	3	1
B	1	3

$3-1 = 2 \to 60$ days
$1 \to 30$
$3 \to 90$

A → 30 D
B → 90 D
90 → 3 : 1
$A+B = \frac{90}{4} = 22.5$ D

$A = x$ days
$B = 3x$ days
$3x - x = 2x \to 60$ days
$x = 30, 3x = 90$
$\frac{1}{30} + \frac{1}{90} = \frac{4}{90} \Rightarrow \frac{90}{4} = 22.5$ D

③ A is twice as fast as B and do the work in 20 days less than B. A+B = ? ($\frac{40}{3}$ Days)

$A = x$ days
$B = 2x$ "
$2x - x = 20$
$x = 20$

A — 20
B — 40
40 : 2 : 1
$A+B = \frac{40}{3}$ days

④ A → 6 days, B → 8 days, C takes as much time as A & B take together. Find time taken by B+C = ? ($\frac{12}{5}$ Days)

$A+B = \frac{1}{6} + \frac{1}{8} \to \frac{24}{7}$ days $= C$

$B+C = 8\frac{1}{8} + \frac{7}{24} = \frac{3+7}{24} = \frac{10}{24} \to \frac{24}{10} = \frac{12}{5}$ Days

⑤ A takes as much time as B & C take together. (40 Days)
A+B = 10 days, C = 20 days. B alone = ?

A = B+C → A+B+C = $\frac{1}{10}+\frac{1}{20}=\frac{3}{20}$ = 2 A's

A's 1 day work = $\frac{3}{40}$, B's 1 d work = $\frac{1}{10}-\frac{3}{40}=\frac{1}{40}$ = 40 Days

⑥ B takes three times as long as A+C together, C takes twice as long as A+B together. A+B+C = 10 days. A alone, B alone, C alone? (24, 40, 30)

3 B = A+C → 3B+1 B = A+B+C

4 B's one day work = (A+B+C)'s 1 day work = $\frac{1}{10}$

B's 1 day work = $\frac{1}{40}$ = 40 D

Also 2 C's 1 day work = (A+B)'s 1 day work

3 C's 1 day work = $\frac{1}{10}$ (adding C both sides)

C's 1 day work = $\frac{1}{30}$ = 30 D

A = $\frac{1}{10}-\frac{1}{30}-\frac{1}{40}=\frac{1}{24}$ = 24 D

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⑦ A and B together → 12 days, B and C together → 16 days. A worked at it for 5 days, B for 7 days and then C takes up and finishes it in next 13 days. How long will C take to finish whole work alone? (24 days)

→ A's 5 day work + B's 7 days work + C's 13 days work = 1

(5 + 2) (2 + 11)

(A+B)'s 5 day work + (B+C)'s 2 day work + C's 11 day work = 1

$\frac{5}{12}+\frac{2}{16}$ + C's 11 day work = 1

C's 11 day work = $1-\frac{5}{12}-\frac{1}{8}=\frac{11}{24}$

C's 1 days work = $\frac{11}{24}\times\frac{1}{11}=\frac{1}{24}$ → 24 days

A Leaves——

① A → 25 days, B → 20 days. worked together for 5 days and then A left. In how many days will B finish work? (11 D)

A – 25, B – 20 ⟩ 100 — 4 × 5 = 20, 5 × 5 = 25, 45

Rem = 100 – 45 = 55 done by B in $\frac{55}{5}$ = 11 Days

② A → 30 days. After 3 days, A leaves & rem. work done by B in 18 days. Time taken by B alone to do all work? (20 D)

All 3 day work = $\frac{3}{30} = \frac{1}{10}$, Rem. = $\frac{9}{10}$

$\frac{9}{10}$ work by B in – 18 days

1 —— B —— $18 \times \frac{10}{9} = 20$ D

③ A → 40 days, B → 50 days. Begin together and after some time A leaves off. Then B finishes rem. work in 20 days. After how many days did A leave? ($\frac{40}{3}$ Days)

A – 40, B – 50 ⟩ 200 — 5, 4 × 20 = 80

Rem. → 120 done by A+B in $\frac{120}{9} = \frac{40}{3}$ days

④ A+B = 30 days. worked together for 20 days then B left A did rem. work in 20 days. A alone = ? B alone = ? (60, 60)

B left after 20 days. Rem. days = 30 – 20 = 10 days – done by A in 20 D

So A does 1 unit in 2 days

—— 30 —— 60 days.

A+B – 30, A – 60 → B = $\frac{60}{1}$ = 60 D

⑤ A → 20 days, B → 30 days. Start together but B leaves before 5 Days of Completion. Total time to complete work? (14 days)

A – 20, B – 30 ⟩ 60 — 3 × 5 = 15, 2

Rem. = 45 done by A+B in $\frac{45}{5} = 9$ days

Total = 9 + 5 = 14 days

⑥ A → 10 days, B → 12 days, C → 15 days. Started together.
A leaves after 2 days & B leaves before 3 days of completion.
Total time to complete work? (7 days)

A–10, B–12, C–15 → 60 → 6×2 = 12, 5×2 = 10, 4×2 = 8 → 30

4×3 = 12

Rem = 60 − 30 − 12 = 18 done B+C in $\frac{18}{9}$ = 2 days.

Total = 2 + 2 + 3 = 7 days

⑦ A → 8 days, B → 16 days, C → 24 days. Started together.
A continues till finish, C left before 2 days and B left before 1 day of its completion. Total time to finish? (5 Days)

A–8, B–16, C–24 → 48 → 6×2 = 12, 3×1 = 3, 2 → 15

Rem. → 48 − 15 = 33

done by A+B+C in $\frac{33}{6+3+2}$ = 3 days

Total = 3 + 2 = 5 days

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⑧ A started a work and left after 2 days. Then B was called and finished it in 9 days. Had A left after 3 days, B would finish it in 6 days. A alone = ?, B alone = ? (5, 15 days)

Let A = x days, B = y days.

$\frac{2}{x} + \frac{9}{y} = 1$ Put $\frac{1}{x} = a$, $\frac{1}{y} = b$ $2a + 9b = 1$ $a = \frac{1}{5}$ so $x = 5$ days

$\frac{3}{x} + \frac{6}{y} = 1$ $3a + 6b = 1$ $b = \frac{1}{15}$ $y = 15$ days

OR

Inc in 1 day of A = dec. in 3 days of B

A = 3B

A – 3 units × 2 → 6 units in 2 days

B – 1 unit × 9 → 9 units in 9 days

Total 15 units so A = $\frac{15}{3}$ = 5 days

B = $\frac{15}{1}$ = 15 days

WORK & WAGES

① A = 6 days, B = 8 days, C = 12 days. Total ₹ 1350. find share of B? (450)

A : B : C
$\frac{1}{6} : \frac{1}{8} : \frac{1}{12}$
4 : 3 : 2

or

A-6, B-8, C-12 → 24 → 4, 3, 2 → 4 : 3 : 2

② A = 6 days, B = 8 days. Total ₹ 200. with the help of C they can do it in 3 days. Find share of C? (25)

C → $1-(\frac{3}{6}+\frac{3}{8}) = \frac{1}{8}$

A : B : C
$\frac{3}{6} : \frac{3}{8} : \frac{1}{8}$
4 : 3 : 1

or

A-6, B-8, A,B,C-3 → 24 → 4, 3, 8

C = 8-(4+3) = 1
so A : B : C
4 : 3 : 1

③ A, B & C contract a work for ₹ 550. A and B do $\frac{7}{11}$ of work. How much does C get? (200)

$C = 1-\frac{7}{11} = \frac{4}{11}$ (A+B) : C

$\frac{7}{11} : \frac{4}{11}$ → 7 : 4 → $C = \frac{4}{11} \times 550 = 200$

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④ 3M + 4B can earn ₹ 756 in 7 days. 11 M + 13 B can earn ₹ 3108 in 8 days. In what time will 7 M + 9 B earn ₹ 2480? (10 D)

(3M+4B) in 1 day — $\frac{756}{7} = 108$ ₹, 11M+13B in 1 day — 376 ₹

$\frac{3M+4B}{108} = \frac{11M+13B}{376} = 1$ ₹ / 1 day → 3M = 5B

so $3M + 4 \times \frac{3}{5}M \rightarrow \frac{27}{5}M$ in 1 day earn ₹ 108

1 M in 1 day earn $\frac{108 \times 5}{27} = 20$ ₹

Daily earning of a man = 20, Daily earning of boy = $\frac{3}{5} \times 20 = 12$

7M + 9B earn ₹ (7×20 + 9×12) = 248 ₹ in 1 day

2480 ₹ in — 10 days

PIPES & CISTERNS

① Pipe A = +20 hrs, Pipe B = +30 hrs, Pipe C = −40 hrs. How much time is needed to fill tank if all work together? ($\frac{120}{7}$ hrs)

A → 20, B → 30, C → −40 ⟩ 120 ⟨ 6, 4, −3

$\frac{120}{6+4-3} = \frac{120}{7}$ hrs

② Pipe A = 15 hrs but due to leak in bottom of tank, it is filled in 20 hrs. if tank is full, how much time will leak take to empty it? (60 hrs)

A → 15, A+B → 20 ⟩ 60 ⟨ 4, −3

B = 4 − 3 = 1

$\frac{60}{1}$ = 60 hrs

③ Pipe A = 24 mins, Pipe B = 32 mins. Both opened simult. After how much time should A be closed so that tank is filled in just 16 mins? (12 mins)

$\frac{n}{24} + \frac{16}{32} = 1 \rightarrow n = 12$ mins

A − 24, B − 32 ⟩ 96 ⟨ 4, 3 × 16 = 48

Rem → 96 − 48 = 48 by A in $\frac{48}{4}$ = 12 mins

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④ A+B+C → 6 hrs. After working together for 2 hrs, C is closed and A+B fill rem. tank in 8 hrs. C alone? (12 hrs)

A+B+C fill in 2 hrs = $\frac{2}{6} = \frac{1}{3}$rd Rem. → $\frac{2}{3}$ filled by A+B in 8 hrs.

$\frac{8 \times 3}{2}$ = 12 hrs

A+B+C — 6, A+B — 12 ⟩ 12 ⟨ 2, 1

C = 2 − 1 = 1 → $\frac{12}{1}$ = 12 hrs

⑤ A tank has a leak which would empty it in 8 hrs. A tap is turned on which admits 6 ltrs/min into tank and it is now ~~filled~~ emptied in 12 hrs. Find capacity of tank? (8640 ℓ)

A → 8, A+B → 12 ⟩ 24 ⟨ 3, 2

B = $\frac{24}{1}$ = 24 hrs

Capacity = 24 × 60 × 6 = 8640 ltrs

PERMUTATION — COMBINATION — PROBABILITY

* Permutation ← Combination → selection

Permutation → Arrangement

$^nP_r \rightarrow \frac{n!}{(n-r)!}$

$^nC_r = \frac{n!}{r!(n-r)!}$

Factorial

ex → 6! → 6×5×4×3×2×1

11! → 11×10×9×8×7!

Permutation Cases → formation of words with given letters, formation of numbers with given digits, arrangement of books on shelf etc.

Combination Cases → formation of Committee, formation of team.

[Q1] How many ways can the letters of 'CIVILIZATION' be arranged?

CIVILIZATION
1 2 3 4 5 6 7 8 9 10 11 12

$\frac{12!}{4!}$ → I is repeated 4 times

[Q2] PATALIPUTRA

P → 2
T → 2
A → 3

$\frac{11!}{2! \times 3! \times 2!}$ Ans.

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[3] TRIANGLE → All the vowels always come together.?

TRNGL (IAE)
5 1
6

$6! \times 3!$ → Three vowels can shuffle in 3! ways.

[4] MATHEMATICS (All vowels come together) ?

MTHMTCS (AEAI)
7 1

$\frac{8! \times 4!}{2! \times 2! \times 2!}$

M → 2
T → 2
A → 2

Combination

[5] There are 3 teachers, 4 doctors and 6 engineers. A team of 5 members is to be formed. Find no. of ways if:-

(i) There are 2 teachers and 3 engineers → $^3C_2 \times {}^6C_3$ → 60

(ii) There are 4 doctors & 1 engineer or 3 teachers & 2 doctors

$(^4C_4 \times {}^6C_1) + (^3C_3 \times {}^4C_2)$ → 12

(iii) Atleast one engineer → Total − No engineer

$[^{13}C_5 - {}^7C_5]$ Ans

* Probability = $\frac{\text{No. of favorable outcomes}}{\text{Total No. of outcomes}}$

[Note:- Combination Concept is used in Probability]

* Total outcomes in different cases

1 coin → 2^1 (H,T) 2 coins → 2^2 → 4 (HT, TT, TH, TT), 3 coins → 2^3 → 8

1 dice → 6 (1,2,3,4,5,6), 2 Dice → $6^2 = 36$

1 card from pack → 52, 2 cards → ${}^{52}C_2$, 3 cards → ${}^{52}C_3$

[Note:- if 1 object choosen then ${}^nC_1 \to n$]

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* Cards → (52)

4 king
4 queen
4 Jacks
12 Face cards

Black (26) — Club (13), Spade (13)
Red (26) — Diamond (13), Heart (13)

Q① A dice is rolled. Find Probability of getting an even no.?

→ Favorable → 3 (2,4,6) $P = \frac{3}{6} = \boxed{\frac{1}{2}}$
Total → 6

Q② A letter is selected at random from 'PROBABILITY'. Find the prob. that it is a vowel?

→ Favorable → 4 (OAII) $\boxed{P = \frac{4}{11}}$
Total → 11

Q③ Find the Prob. that a leap year selected at random has 53 sundays? → $\frac{366}{7}$ → 52 (each day) + 2 Rem.

(S,M) (M,T) (T,W) (W,T) (T,F) (F,S) (S,S)

Favorable → 2 [(S,M) (S,S)]
Total → 7

$\boxed{P = \frac{2}{7}}$ Ans.

Probability

Q① When two dice are thrown what is the prob. that:-

(1,1) (1,2) ---- (1,6)
(2,1) (2,2) ---- (2,6)
(3,1) (3,2) ---- (3,6)
(4,1) (4,2) ---- (4,6)
(5,1) (5,2) ---- (5,6)
(6,1) (6,2) ---- (6,6)

(I) sum of nos is 6. → $\left[\frac{5}{36}\right]$

(II) sum of nos is ≤ 8 → $\left[\frac{26}{36}\right]$

(III) sum of nos is a multiple of 3 → $\left[\frac{12}{36}\right]$

(IV) Nos shown are equal → $\left[\frac{6}{36}\right]$

(V) Diff of the nos is 2 → $\left[\frac{8}{36}\right]$ → (3,1), (4,2), (5,3) (6,4) (4,6) (3,5) (2,4) (1,3)

Q② A Card is drawn from a pack of cards. what is the prob.:-

(1) A black suit card → 26/52

(2) Card having no. less than 7 → $\left[\frac{5\times4}{52} \to \frac{5}{13}\right]$ → (2,3,4,5,6) not ikka

(3) A king or a queen → $\left[\frac{1}{13}+\frac{1}{13} \to \frac{2}{13}\right]$

(4) A king or a club → $\left\{\frac{4+13-1}{52}\right] \to \frac{16}{52} \to \frac{4}{13}\right]$

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Q③ From a pack of 52 cards, 3 cards are drawn. find prob. that:-

Total outcome: $^{52}C_3 \to \frac{52\times51\times50}{3\times2\times1} = 26\times17\times50$

(I) All three are Jacks. $\frac{^4C_3}{^{52}C_3} \to \frac{4}{26\times17\times50} \to \boxed{\frac{1}{5525}}$

(II) One ace, 1 king and 1 queen $\frac{4\times4\times4}{26\times17\times50} = \boxed{\frac{16}{5525}}$

(III) one king and 2 Jacks $\frac{^4C_1\times{}^4C_2}{^{52}C_3} \to \frac{4\times6}{^{52}C_3} \to \boxed{\frac{6}{5525}}$

(IV) 2 digit cards and 1 ~~face~~ honor card of red color $\frac{^{36}C_2\times{}^8C_1}{^{52}C_3} \to \frac{18\times35\times8}{26\times17\times50} \to \boxed{\frac{252}{1105}}$

Q④ A Bag contains 3R + 5Y + 4G Balls. 3 balls are drawn randomly, find prob.;

Total: $^{12}C_3 \to \frac{12\times11\times10}{6} = 220$

(I) balls of diff colors → $^3C_1\times{}^5C_1\times{}^4C_1 \to \frac{3\times5\times4}{220} = \boxed{\frac{3}{11}}$

(II) exactly 2 G Balls → $^4C_2\times{}^8C_1 \to \frac{6\times8}{220} = \boxed{\frac{12}{55}}$

(III) no yellow ball → $^7C_3 \to \frac{35}{220} = \boxed{\frac{7}{44}}$

(IV) all are of same color → $^3C_3 + {}^5C_3 + {}^4C_3 \to \frac{1+10+4}{220} = \frac{15}{220} = \boxed{\frac{3}{44}}$

Q(5) 5 B + 7 W balls.
A ball is drawn out and replaced in bag. Then a ball is drawn again. Find prob. that:-

(I) Both were black → $5/12 \times \frac{5}{12} \rightarrow \boxed{\frac{25}{144}}$

(II) 1st white, 2nd black → $\frac{7}{12} \times \frac{5}{12} = \boxed{\frac{35}{144}}$

~~Q(6) 4W + 6B balls.~~

Q(6) 4 Blue + 2 Green + 3 Red Marbles
Two are drawn randomly. Find the prob. that:-

Total → ${}^9C_2 \rightarrow \frac{9 \times 8}{2 \times 1} = 36$

(I) Both Green → $\frac{{}^2C_2}{36} \rightarrow \boxed{\frac{1}{36}}$

(II) 1 Blue + 1 Red → $\frac{{}^4C_1 \times {}^3C_1}{36} = \frac{4 \times 3}{36} = \boxed{\frac{1}{3}}$

(III) both blue or both red → $\frac{{}^4C_2 + {}^3C_2}{36} = \frac{9}{36} = \boxed{\frac{1}{4}}$

(IV) None is blue → $\frac{{}^5C_2}{36} = \boxed{\frac{5}{18}}$

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(V) At least one Green → 1 − None Green → $1 - \frac{{}^7C_2}{36} = 1 - \frac{7}{12} = \boxed{\frac{5}{12}}$

Q(7) 5 MEN + 4 WOMEN
A Committee of 4 persons to be selected. Find the prob. that:-

[Total → ${}^9C_4 \rightarrow \frac{9 \times 8^2 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126$]

(I) All are women → $\frac{{}^4C_4}{126} = \boxed{\frac{1}{126}}$

(II) All are men → $\frac{{}^5C_4}{126} = \boxed{\frac{5}{126}}$

(III) 2 Men + 2 women → $\frac{{}^5C_2 \times {}^4C_2}{126} = \frac{60}{126} = \boxed{\frac{30}{63}}$

(IV) Atleast 1 woman → 1 − NO woman
$= 1 - \frac{5}{126} = \boxed{\frac{121}{126}}$

MISCELLANEOUS

Q1 A person spent 28% of his income on food, 50% of the remaining on housерent and 75% of further remainder on other items. He is now left with ₹ 11700. Find his income?

→ Initial value = Final value × $\frac{100}{100-x\%}$ × $\frac{100}{100-y\%}$ × $\frac{100}{100-z\%}$ - - - .

$= 11700 \times \frac{100}{72} \times \frac{100}{50} \times \frac{100}{25} \Rightarrow$ **1,30,000**

Q2 If Numerator of a fraction is increased by 250% and denominator is increased by 400%, then fraction becomes 7/17. Find original fraction?

→ $\frac{x+2.5x}{y+4y}$ or $\frac{100x+250x}{100y+400y} = \frac{7}{17} \Rightarrow \frac{350x}{500y} = \frac{7}{17} \Rightarrow \frac{x}{y} = \frac{10}{17}$

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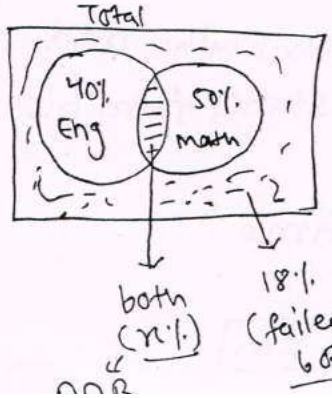
Q3 If the price of sugar increases by 20%, then by how much percent must a householder decrease his consumption so as not to increase his expenditure?

→ 100 $\xrightarrow{20\%\uparrow}$ 120 Now 120 ⟶ 100 $\frac{20}{120} \times 100 = 16\frac{2}{3}\%$

Q4 In a class, 40% students passed in Eng and 50% passed in math. If 18% failed in both Eng and math, then find percentage of students passed in both?

→ 

$A \cup B = A + B - A \cap B$

$100 - 18 = 40 + 50 - x$

$82 = 90 - x$

$x = 8\%$ Passed in both

Q5 In an exam, A scored 25% marks & failed by 15 marks. B scored 32% and got 20 marks more than passing marks. Find Max. Marks for the exam?

→ $25\% + 15 = 32\% - 20$
Passing mks Passing mks

$7\% = 35$
$1\% = 5$
$100\% = 500$

Q6 A reduction of 20% in the price of rice enables a person to buy 3.5 kg more rice for ₹ 385. Find the original price and new price of rice?

→ Let original price = ₹ x/kg
New price = $\frac{80}{100}x = \frac{4}{5}x$ /kg

$\frac{385}{4x} \times 5 - \frac{385}{x} = 3.5$

$x = 27.5$ = original

$\frac{4}{5} \times 27.5 = 22$ = New

Shortcut

New Price = $\frac{20\% \text{ of } 385}{3.5}$

$= \frac{77}{35} \times 10^2 = 22$ /kg

Original : New
~~100~~ : ~~80~~
5 : 4

original = $\frac{5}{4} \times 22$
= 27.5 /kg

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Q7 A man goes from his home to office @ 30 km/hr and gets late by 10 min. Next time, he goes @ 40 km/hr but now he reaches 5 min earlier. Find Distance b/w home & office?

→ 30 → +10 min
x
40 → −5 min
15 min.

$\frac{x}{30} - \frac{x}{40} = \frac{15}{60}$ hr. ⇒ $x = 30$ km

Shortcut $\frac{S_1 \times S_2}{S_1 - S_2} \times$ time diff.

$\frac{40 \times 30}{10} \times \frac{15}{60} = 30$ km

Q8 A man goes from his house to office @ 25 km/hr and returned @ 4 km/hr. He took $5\frac{4}{5}$ hrs. Find Distance from his house to office?

→ $\frac{x}{25} + \frac{x}{4} = \frac{29}{5}$

$x = 20$ km

Shortcut $\frac{S_1 \times S_2}{S_1 + S_2} \times$ total time

$\frac{25 \times 4}{25 + 4} \times \frac{29}{5} = 20$ km

Q9 Speeds of A and B are in ratio 3:4. A takes 40 mins more than B to reach a place. How much time will A take to reach that place?

→

	A	B
Speeds →	3	4
time →	4	3

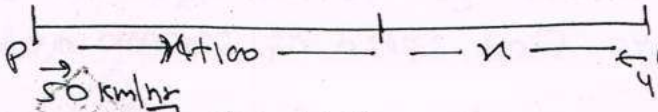
Speed $\propto \frac{1}{\text{time}}$

1 → 40 min

4 → 160 min → **2 hr, 40 min** = Time taken by A

3 → 120 min → **2 hrs** = Time taken by B.

Q10 Two trains start from P & Q & travel towards each other @ 50 km/hr & 40 km/hr resp. By the time they met the first train had travelled 100 km more than second. Find distance b/w P and Q?



→

$t_1 = t_2$

$\frac{x+100}{50} = \frac{x}{40}$ SANDEEP BOHAY™

$x = 400$

Distance PQ → (400+100) + 400 = 900 km

Shortcut

Extra distance × $\frac{S_1+S_2}{S_1-S_2}$

$100 \times \frac{90}{10} = 900$ km

Q11 A train reach a station at a certain time and at a fixed speed, if train had been 10 km/hr faster, it would have taken 2 hrs less than actual time. and if the train were slower by 2 km/hr, it would have taken 3 hrs more than actual time. Find distance of journey & and actual speed?

→

$S_1T_1 = S_2T_2$

$ST = (S+10)(T-2)$

$ST = ST - 2S + 10T - 20$

$2S - 10T = -20$ — (1)

$ST = (S-12)(T+3)$

$ST = ST + 3S - 12T - 36$

$-3S + 12T = -36$ — (2)

Solve (1) & (2)

Shortcut

Actual speed = $\frac{S_1 \times S_2 \times (T_1+T_2)(T_1-T_2)}{T_1 \times T_2}$

$= \frac{10 \times 12 \times 5 \times 1}{6} = 100$ km/hr

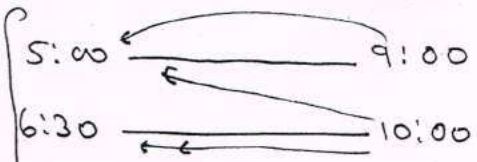
Increased = 110

Decreased = 88

Distance = $\frac{110 \times 88}{110-88} \times 5 = 2200$ km

Q12 A train leaves Patna at 5 a.m. and reaches Delhi at 9 a.m. Another train leaves Delhi 6:30 a.m. and reaches Patna at 10 a.m. At what time do the two trains meet?

→



Shortcut

$$\text{Initial time} + \frac{(9:00-5:00)\times(10:00-5:00)}{(9:00-5:00)+(10:00-6:30)}$$

$$5:00 + \frac{4\times5}{4+3.5} \Rightarrow 5:00 + \frac{20\times10^{2}}{75_{3}} \rightarrow 2\frac{2}{3}\text{ hr}$$

$$5:00 + 2\frac{2}{3}\times60 \Rightarrow 5:00 + 2\text{hr } 40\text{min}$$

7:40 ~~min~~ Ans

Q13 A boat goes 30 km upstream and 44 km downstream in 10 hrs. It can go 40 km upstream and 55 km downstream in 13 hrs. Find speed of stream and speed of boat?

→

$$\frac{30}{U} + \frac{44}{D} = 10$$

$$\frac{40}{U} + \frac{55}{D} = 13$$

Solving these 2 eqns we get $U = 5$ (upstream speed), $D = 11$ (Downstream speed)

Speed of boat $= \frac{D+U}{2} = \frac{11+5}{2} = 8$ km/hr

Speed of stream $= \frac{D-U}{2} = \frac{11-5}{2} = 3$ km/hr.

Shortcut

5×6 $\quad 11\times4$ $\quad 6+4=10$

$$\frac{30}{U} + \frac{44}{D} = 10$$

5×8 $\quad 11\times5$ $\quad 8+5=13$

$$\frac{40}{U} + \frac{55}{D} = 13$$

$U=5$, $D=11$

Boat → $\frac{11+5}{2} = 8$ km/hr

stream → $\frac{11-5}{2} = 3$ km/hr

Q14 Without stoppage, the speed of a bus is 54 km/hr and including stoppage, it is 45 km/hr. For how many minutes does the bus stop per hour?

→ Shortcut →

$$\frac{\text{without stoppage} - \text{with stoppage}}{\text{without stoppage}} \Rightarrow \frac{9}{54}\ \frac{1}{6}\text{ hr} \times 60$$

10 mins

MATHS NOTES (2) BY SANDEEP BOHAY

GEOMETRY

- ✓ LINES,ANGLES,POLYGONS
- ✓ TRIANGLES & QUADRILATERALS
- ✓ CIRCLES

TRIGONOMETRY

- ✓ TRIGONOMETRIC RATIOS
- ✓ IDENTITIES
- ✓ MAXIMUM & MINIMUM VALUES
- ✓ HEIGHT & DISTANCE

ALGEBRA

- ✓ LINEAR EQUATIONS
- ✓ COORDINATE GEOMETRY
- ✓ POLYNOMIALS
- ✓ ADVANCED ALGEBRA

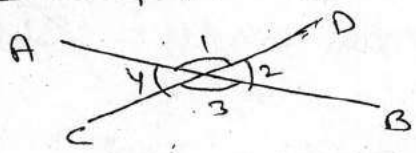
MENSURATION

- ✓ 2D MENSURATION
- ✓ 3D MENSURATION

* Supplementary Angles $\alpha + \beta = 180^\circ$ (सम्पूरक)

* Complimentary Angles $\alpha + \beta = 90^\circ$ (पूरक)

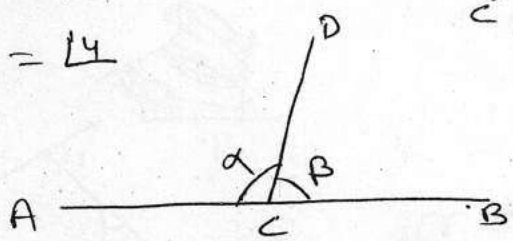
* Vertically opp. Angles (V.O.P.) (शीर्षाभिमुख)



$\angle 1 = \angle 3$

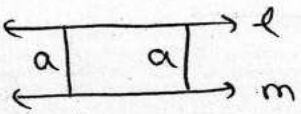
$\angle 2 = \angle 4$

*



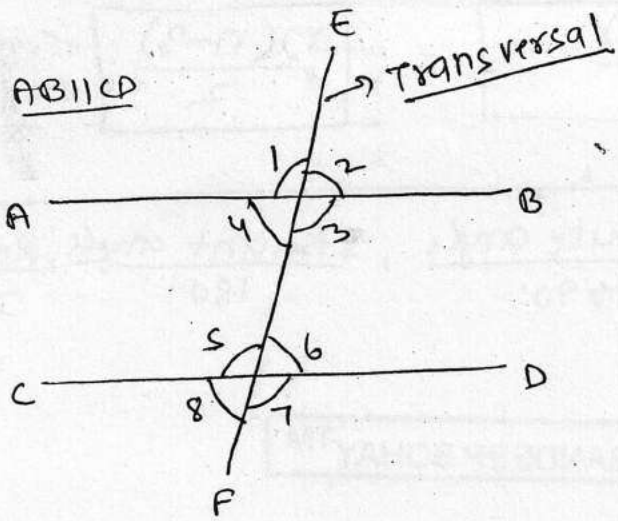
$\alpha + \beta = 180^\circ$

* Parallel lines समानान्तर



$\ell \parallel m$

* AB || CD



$\angle 2 = \angle 8$
$\angle 1 = \angle 7$
$\angle 3 = \angle 5$
$\angle 4 = \angle 6$

Alternate एकान्तर

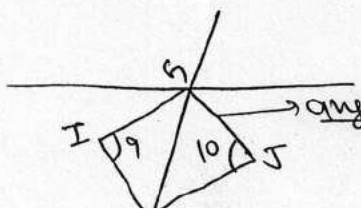
$\angle 1 = \angle 5$
$\angle 2 = \angle 6$
$\angle 3 = \angle 7$
$\angle 4 = \angle 8$

corresponding angles संगत

$\angle 3 + \angle 6 = 180^\circ$

$\angle 4 + \angle 5 = 180^\circ$

[Consecutive interior angles]



→ कोणों के समद्विभाजक

→ $\angle 9 = \angle 10 = 90^\circ$

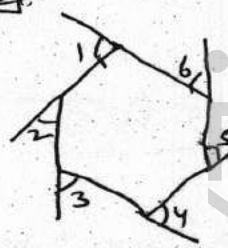
* RE

→ Sum of internal angles = $(n-2)\times 180^\circ = 2(n-2)\times 90^\circ$ अंतः कोण

→ each internal angle = $\frac{(n-2)\times 180^\circ}{n} = \frac{2n-4}{n}\times 90^\circ$

→ Sum of all external angles = 360°

→ each external angle = $\frac{360^\circ}{n}$ बहिष्य कोण



$\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6=360^\circ$

[All extended in one direction]

No. of sides = No. of angles = n

→ Interior angle + exterior angle = 180°

→ No. of Diagonals = $\frac{n(n-1)}{2}-n = \frac{n(n-3)}{2}$ विकर्ण

---

* Right Angle (90°), Acute angle ($<90^\circ$), Obtuse angle ($>90^\circ$), straight angle (180°), Reflex ang ($>180^\circ$)

Angle bisector

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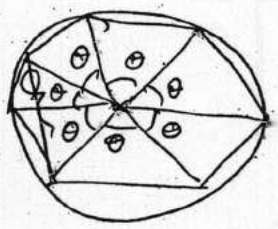
→ 

each equal side of a polygon (regular) subtend equal angle at centre

So $n\theta = 360$

Q① In

$\left[120^\circ = \frac{(n-2)\times 180}{6} \quad \frac{(n-2)\times 180}{n} \Rightarrow 120n = 180n - 360 \rightarrow n = 6\right]$

② Int. angle of a Regular Polygon is 135°. Find no. of sides? (8)

$\left[135 = \frac{(n-2)\times 180}{n} \rightarrow n = 8\right]$

③ The sum of all int. angles of a Regular Polygon is 540. Find the no. of sides? (5)

$\left[540 = (n-2)\times 180 \rightarrow n = 5\right]$

④ The sum of all int. angles of a Regular Hexagon is equal to how many right angles? समकोण (8)

$\left[(6-2)\times 180 = 720^\circ \Rightarrow 720/90 = 8 \text{ right angles}\right]$

⑤ Each exterior angle of a Polygon is 72°. Find no. of sides? (5)

$\left[\frac{360}{72} = 5\right]$

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⑥ Find no. of sides of the Regular Polygon if measure of each angle is

(A) 160° → 18

(B) 108° → 5

(C) 162° → 20

(D) 175° → 72

QUANT PATTERN IN SSC EXAMS

1. GEOMETRY (5-6 Q)
2. TRIGONOMETRY (5-6)
3. ALGEBRA (4-5 Q)
4. MENSURATION (4-5 Q)
5. NO. SYSTEM (4-5 Q)
6. PROFIT-LOSS (4-5 Q)

35 Questions out of 50 are from these 7 Topics.

Rem. 15 Ques

- Time-Work
- Average
- Percentage
- Time-Distance
- Trains
- Boats

⑦ Ar... angles? (~~108°~~)

$[\theta = 4(90-\theta) \rightarrow \theta = 72°, 18°]$

⑧ Find that angle which is 5 times of its supplementry angle? (150°)

$[\theta = 5(180-\theta) \rightarrow \theta = 150°]$

⑨ Find the angle which is 20° more than its comp. angle? (55°)

$[\theta = 90° - \theta + 20 \rightarrow \theta = 55°]$

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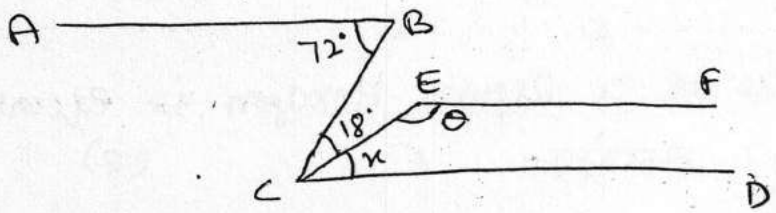
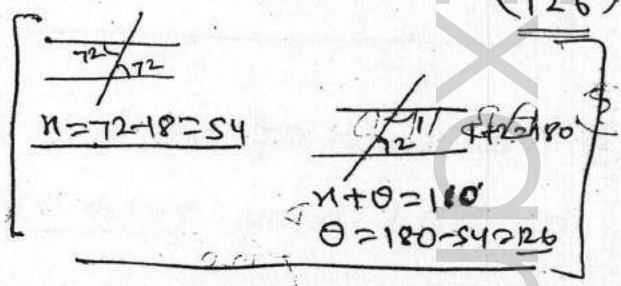
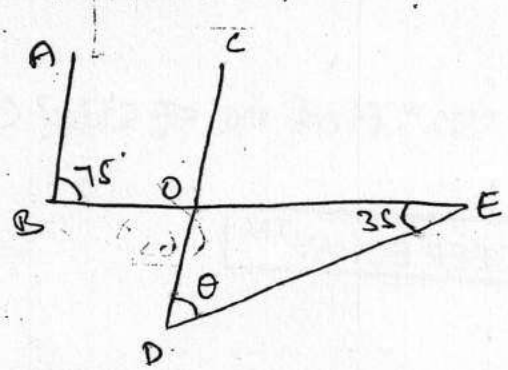
⑩ AB ∥ CD , CD ∥ EF , Find θ = ?



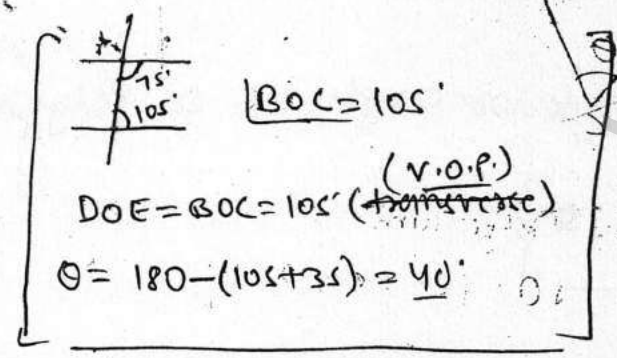
(126°)



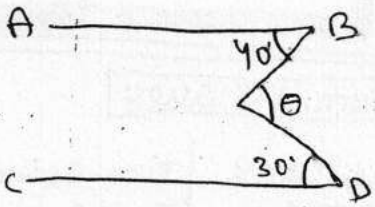
⑪ AB ∥ CD , θ = ?



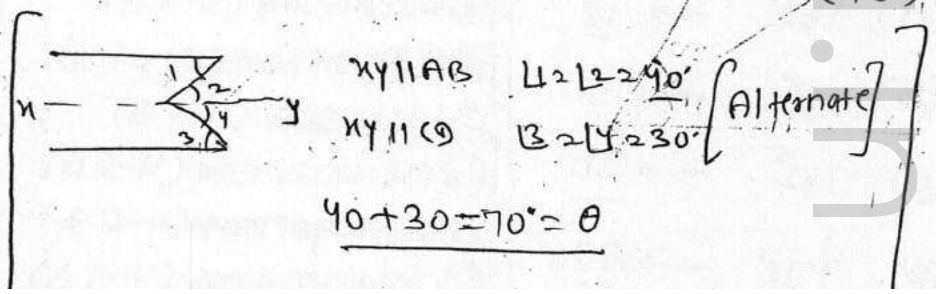
(40°)



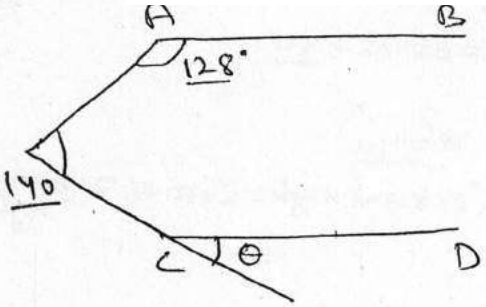
⑫ AB ∥ CD, θ = ?

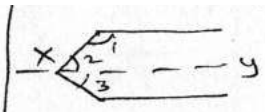


(70°)



(13) AB ?

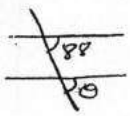




$xy \parallel AB$ $\angle 1 + \angle 2 = 180°$,

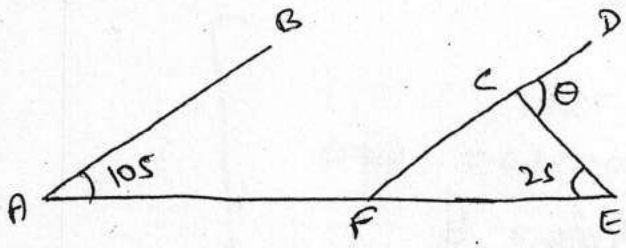
$\angle 2 = 180 - 128 = 52°$, $\angle 3 = 140 - 52 = 88$

$xy \parallel CD$



$\theta = 88°$ = corresponding

(14) $AB \parallel CD$, $\theta = ?$ (130)



$\angle DFE = \angle BAE = 105°$ (corresponding)

$\angle FCE = 180 - (105 + 25) = 50°$

$\theta = 180 - 50 = 130$

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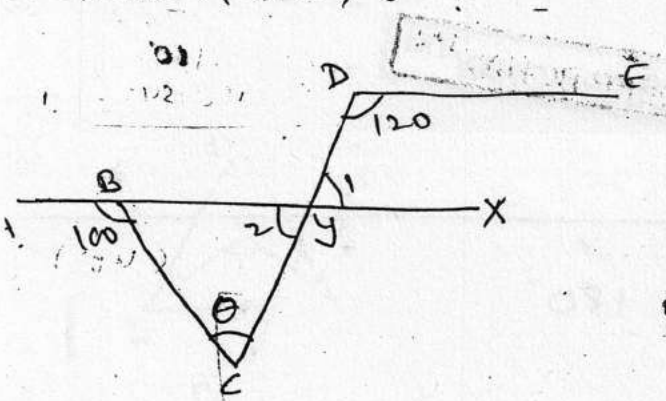
(15) $AB \parallel DE$, $\theta = ?$ (40)



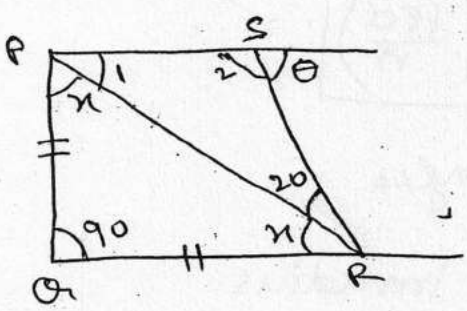
$\angle CBy = 180 - 100 = 80$

$\angle 1 = 180 - 120 = 60 = \angle 2$ (transverse)

$\theta = 180 - (60 + 80) = 40°$

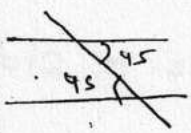
(16) PQRS = Trapezium (समलम्ब चतुर्भुज) (65)

$\times QR \parallel PS$, $PQ = QR$, $\theta = ?$



Corr: angles of equal sides of a $\triangle$ are equal

$2x = (180 - 90) = 90 \rightarrow x = 45°$

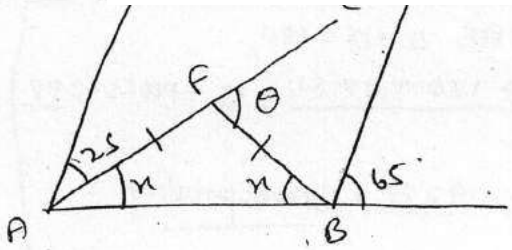


$\angle 1 = \angle x = 45°$ (Alternate)

$\theta = \angle 1 + 20 = 45 + 20 = 65°$

or $\angle 2 = 180 - (20 + 45) = 115°$, $\theta = 180 - 115 = 65°$

(17) A



B ~~P~~ 65 D $x = 65 - 25 = 40$

$AF = BF$ so $x = x$ $x = 40$

$\theta = 40 + 40 = 80$ (external angle = sum of 2 opp int angles)

(18) Ratio of internal angles and ext. angles of a regular Polygon is 2:1. Find no. of sides of Polygon (6)

$$\frac{\frac{(n-2)\times 180}{n}}{\frac{360}{n}} = \frac{2}{1} \rightarrow 180n - 360 = 720$$

$$180n = 720 + 360 = 1080$$

$$n = 1080/180 = 6$$

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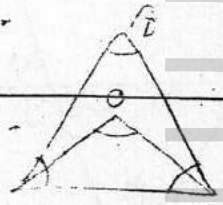
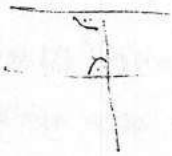


* Convex polygon → all angles $< 180°$

Concave polygon → atleast one angle $> 180°$

Regular polygon → all angles & sides equal.

Area of any polygon = $\frac{n a^2}{4} \cdot \cot\left(\frac{180}{n}\right)$



$n \rightarrow$ no. of sides / angles

2nd formula for Area = $\frac{1}{2} \times$ Perimeter $\times$ inradius.

[Inradius is the perpendicular from centre to any side].

*Types of Triangle on basis of sides

(a) Scalene Δ → All sides different.

(b) Isosceles Δ → Two sides equal समद्विबाहु

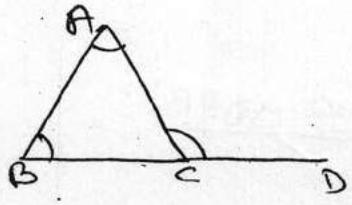
(c) Equilateral Δ → All sides equal समबाहु

* Types of Triangle on basis of angles

(a) Acute angled Δ → each angle acute न्यून कोण

(b) Obtuse angled Δ → one angle obtuse अधिक कोण

(c) Right angled Δ → one angle right angle. समकोण



$\angle ACD = \angle A + \angle B$

and $\angle ACD > \angle A$, $\angle ACD > \angle B$] always

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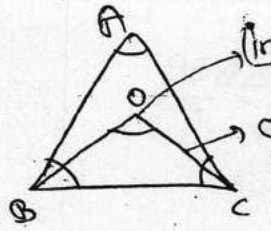
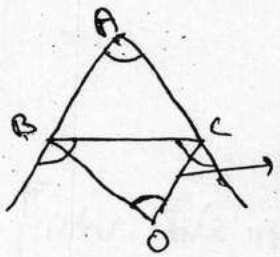


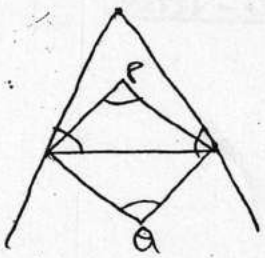
$\angle BOC = 90° + \frac{\angle A}{2}$ always obtuse angle (अधिक कोण)

कोण समद्विभाजक

Angle bisectors



$\angle BOC = 90° - \frac{\angle A}{2}$ always acute angle (न्यून कोण)

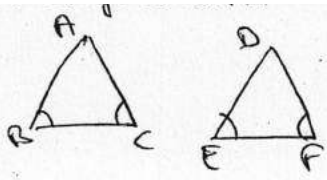


$\angle P + \angle Q = 180°$

** Congruence of two triangles (सर्वांगसम)

(a) Side – side – side (S-S-S)

Th... Sum of ... angles & included side

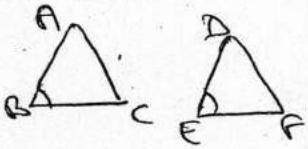


$\angle B = \angle E$ $\quad BC = EF$

$\angle C = \angle F$ $\quad \triangle ABC \cong \triangle DEF$

(c) Side-Angle-Side (S-A-S)

Two sides & included angle.



$AB = DE$ $\quad \angle B = \angle E$

$BC = EF$ $\quad \triangle ABC \cong \triangle DEF$

(d) Angle-Angle-side (A-A-S)

Two angles and non-included side.

$\angle A = \angle D$ $\quad BC = EF$ $\quad \triangle ABC \cong \triangle DEF$

$\angle B = \angle E$

(e) Right angle-Hypotenuse-side (R-H-S)

of

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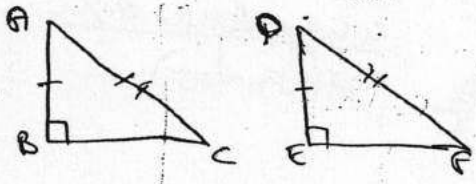
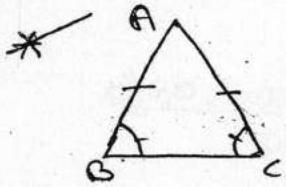


$\therefore \angle B = \angle E = 90°$

$AB = DE$

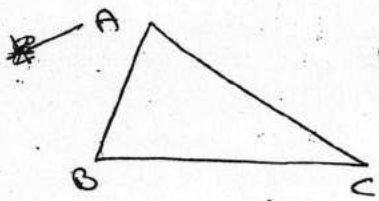
$AC = DF$

$\triangle ABC \cong \triangle DEF$



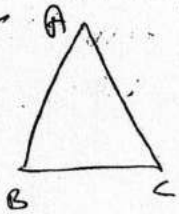
If $AB = AC$

then $\angle B = \angle C$ or vice-versa



if $AC > AB$

then $\angle B > \angle C$ [Angle opp to bigger side will be bigger and vice-versa]



$AB + AC > BC$ $\quad BC \geq AB - AC$

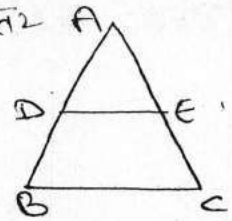
$AB + BC > AC$ and $AC \geq AB - BC$

$AC + BC > AB$ $\quad AB \geq AC - BC$

* Mid

A line segment joining mid pts of any two sides of a △ is parallel to third side and half of it

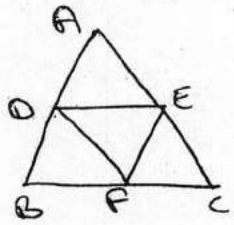
समानान्तर



D is mid pt of AB

E is mid pt of AC

$DE \parallel BC$ and $DE = \frac{1}{2}BC$



D, E & F are mid pts.

then Area of $\triangle DEF = \frac{1}{4}$ Area of $\triangle ABC$

and all small △s have equal area.

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→ And if ABC is equilateral △ then DEF is also equilateral.

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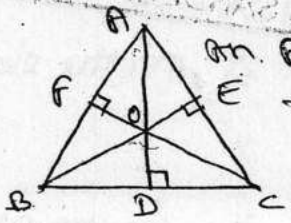
* Altitude (लम्ब)



An Altitude is the ⊥r drawn from one vertex to the opp side.

Altitudes meet at Orthocentre (O)

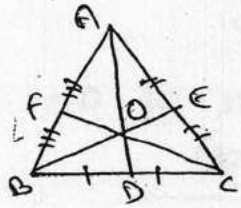
$\angle BOC = 180° - \angle A$

$\angle AOB = 180° - \angle C$

$\angle AOC = 180° - \angle B$

[In quad. AFOE → $\angle E + \angle F = 180°$ (90° each
so $\angle A + \angle O = 180$, $\angle FOE = 180 - \angle A$
so $\angle BOC = 180 - \angle A$ (V.O.A.)]

* MEDIAN (माध्यिका) → A median is the line segment from vertex of a △ to the mid pt of opp side.



All three medians meet at Centroid (O)

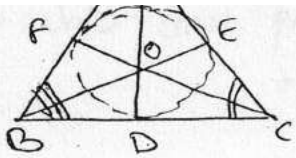
केन्द्रक | माध्य केन्द्र | गुरुत्व केन्द्र

→ Centroid divides any median in ratio 2:1

$\frac{OA}{OD} = \frac{OB}{OE} = \frac{OC}{OF} = \frac{2}{1}$

$OD = \frac{1}{3}AD$, $OE = \frac{1}{3}BE$, $OF = \frac{1}{3}CF$

[Median divides the △ into two △s of equal areas]

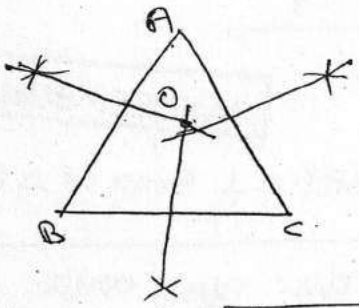


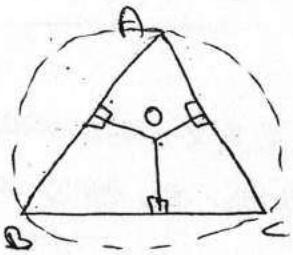
The pt. of intersection of the angle bisectors is called **Incentre** (अन्तः केन्द्र)

$OD = OE = OF$ = radius of incircle.

$$\frac{BD}{CD} = \frac{AB}{AC}$$

* ⊥r bisectors (लम्ब समद्विभाजक)





⊥r bisector meet at **circumcentre** परिकेन्द्र

$OA = OB = OC$ = radius of circumcircle

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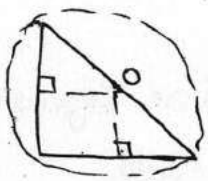
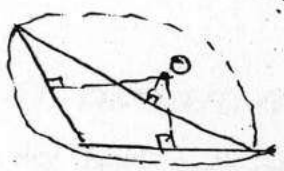
→ Circumcentre of the △ may lie inside △, on the side of △ and outside the △.

→ Circumcentre lies inside the △ if △ is acute angled △

→ Circumcentre lies on the mid pt of hypotenuse of a rt angled △.

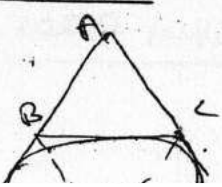
→ Circumcentre lies outside the △ if △ is obtuse angled △.





* Orthocentre, Centroid, Incentre and Circumcentre of an equilateral △ are same points. (lies on one point)

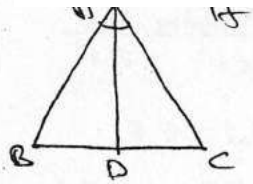
* Excentre बहिःकेन्द्र



The bisectors of two exterior angles of a △ is called excentre. corresponding to the side of the △.

Any △ has three excentres

* Ar

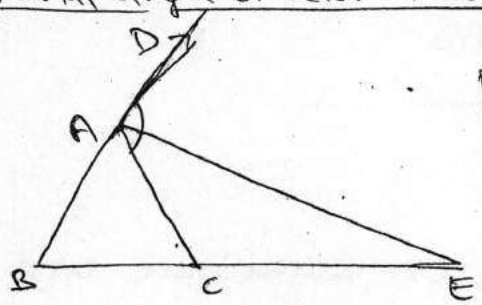


If AD is angle bisector

then $\frac{AB}{AC} = \frac{BD}{DC}$

$AB \times AC - BD \times DC = AD^2$ **

* External angle bisector theorem



$\frac{AB}{AC} = \frac{BE}{CE}$

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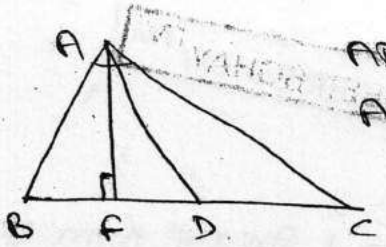
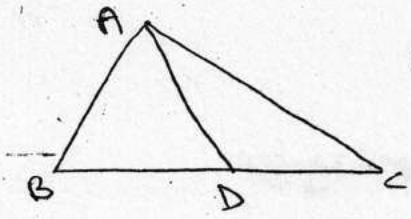


AD = Angle bisector

AF ⊥ BC

then $\angle DAF = \frac{1}{2}(\angle B - \angle C)$

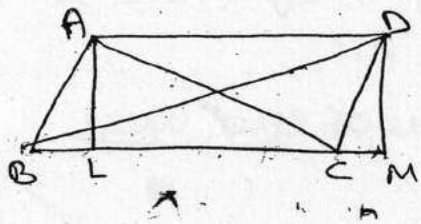


AD = median

Apollonius theorem

$AB^2 + AC^2 = 2(AD^2 + BC^2)$

* Triangles formed on same base (आधार) and between two parallel lines have equal Areas.



BC ∥ AD

Area of △ ABC = Area of △ DBC

If a △ and a parallelogram are ~~formed~~ on same base & b/w same parallels then

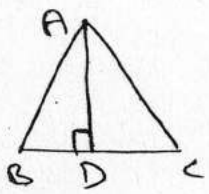
~~Area of △ = $\frac{1}{2}$ Area of ||gm~~

if Area of $\triangle ABC$ = Area of $\triangle DEF$
and $BC = EF$
then $AL = DM$

then $\frac{A(ABC)}{A(DEF)} = \frac{BC}{EF}$

& if $BC = EF$

then $\frac{\triangle ABC}{\triangle DEF} = \frac{AL}{DM}$

* 

Area of $\triangle ABC = \frac{1}{2} BC \times AD$

* if one $\triangle$ and parallelogram (समान्तर चतुर्भुज) lie on same base and between two parallel lines then

Area of $\triangle$ = $\frac{1}{2}$ Area of ||gm

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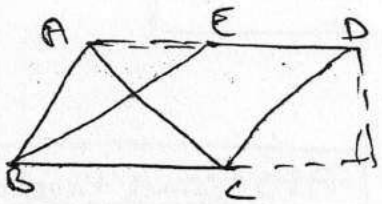
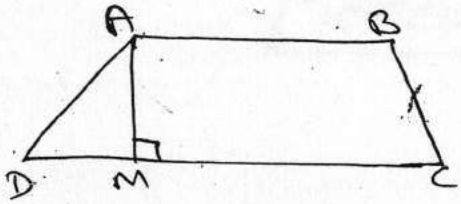


Area of $\triangle ABC = \frac{1}{2}$ Area of ||gm BCDE

* 

Trapezium समलम्ब चतुर्भुज

Area of $ABCD = \frac{1}{2}(AB + CD) \times AM$

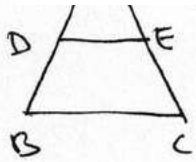
* if two medians (or altitude or ⊥r bisector or Angle bisector) of a $\triangle$ are equal → isosceles $\triangle$

if all three medians of a $\triangle$ are equal → equilateral $\triangle$

* median divides the $\triangle$ into two triangles of equal areas

AD = median

area($\triangle ABD$) = area($\triangle ADC$)



$\frac{AD}{DB} = \frac{AE}{EC}$, $\frac{AD}{BD} = \frac{AE}{CE}$, $\frac{AD}{AB} = \frac{AE}{AC}$

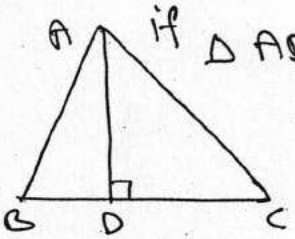
and $\Delta ADE \sim \Delta ABC$ → similar triangles.

---

* if two medians of a triangle are also the angle bisectors then triangle is equilateral Δ.

---

*

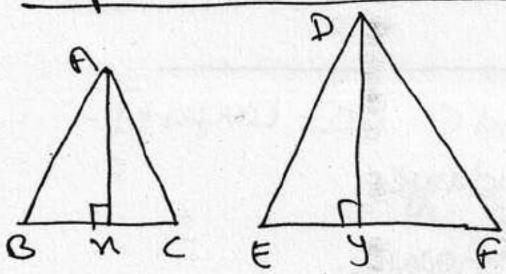


if ΔABC is an equilateral Δ. and $AD \perp^r BC$

then $\frac{AB^2}{AD^2} = \frac{4}{3}$ or $3AB^2 = 4AD^2$

---

* Properties of similar triangles:-



if $\Delta ABC \sim \Delta DEF$ (similar)

then

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AN}{BY} = \frac{\text{Perimeter 1}}{\text{Perimeter 2}}$ etc

and $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \text{- - - - etc.}$

---

* Areas of triangle:-

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① $\frac{1}{2} \times \text{Base} \times \text{height}$

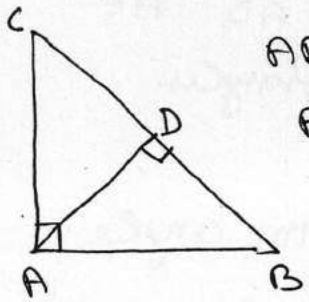
② $\frac{\sqrt{3}}{4} a^2$ → equilateral Δ.

③ $\sqrt{s(s-a)(s-b)(s-c)}$ → any Δ with given sides

where $s = \frac{a+b+c}{2}$

④ $\frac{b\sqrt{4a^2-b^2}}{4}$ → Isosceles Δ

* Pro



ABC is right angle Δ in which $\angle A = 90°$
AD is perpendicular to BC.

① $\text{Area} = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times BC \times AD$

② $AD^2 = BD \times DC$

③ $AB^2 = BC \times BD$
$AC^2 = BC \times CD$

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④ $AD \times BC = AB \times AC$

⑤ $\frac{1}{AD^2} = \frac{1}{AB^2} + \frac{1}{AC^2}$ → Y. Imp.

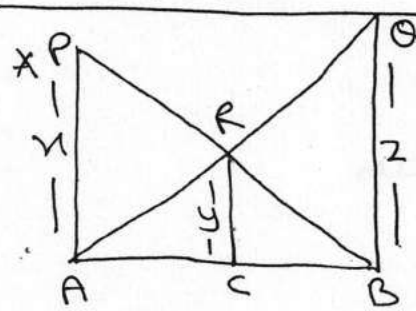
---

* For given three sides a, b and c [a = Largest]
1) if $a^2 = b^2 + c^2$ → Right angle triangle.
2) if $a^2 > b^2 + c^2$ → obtuse angle triangle.
3) if $a^2 < b^2 + c^2$ → Acute angle triangle

---

* For given (equal) perimeter of different triangles, equilateral Δ has maximum area.

* For given (equal) perimeter of diff. quadrilaterals, square has max. area.

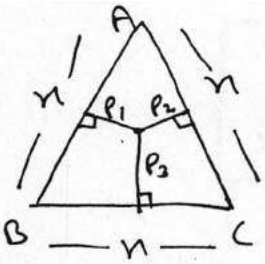


$xy + yz = xz$ —

$\frac{BC}{AB} = \frac{y}{x}$ — ① $\frac{AC}{AB} = \frac{y}{z}$ — ②

① + ② $\frac{AC + BC}{AB} = \frac{xy + yz}{xz}$

$\frac{xy + yz}{xz} = 1$ → $xy + yz = xz$



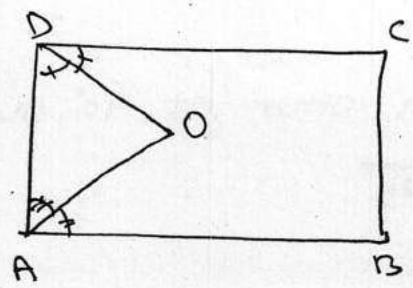
if lengths of perpendiculars drawn from any point in the interior of an equilateral Δ to the sides of Δ are P_1, P_2 and P_3 then side of the Δ is given by.

$$x = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$$

and $$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{3}(P_1 + P_2 + P_3)^2$$

---

*

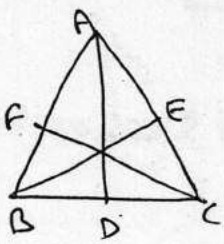


if ABCD is a quadrilateral and OD, OA are angle bisectors then

$$\angle AOD = \frac{(\angle B + \angle C)}{2}$$

---

*

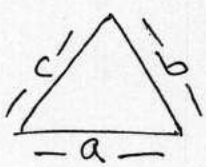


AD, BE, CF → medians

$$AB^2 + BC^2 + CA^2 = \frac{4}{3}(AD^2 + BE^2 + CF^2)$$

---

* Radius of Incircle, i.e. Inradius = $\frac{\text{Area of } \Delta}{\text{semi perimeter}}$



or $$r = \frac{\Delta}{s}$$

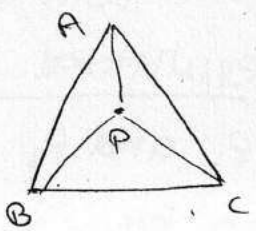
Radius of circumcircle, i.e. circumradius = $\frac{abc}{4 \times \text{Area of } \Delta}$

$$R = \frac{abc}{4\Delta}$$

---

* Distance between Incentre & circumcentre is given by:-

(1) if

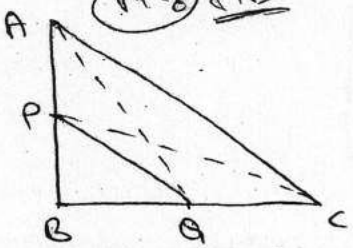


(1) $PA+PB+PC > \frac{1}{2}(AB+BC+AC)$

(2) $AB+BC+AC > PA+PB+PC$

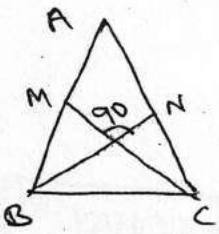
(2) $\angle B = 90°$ and P & Q are mid pts of AB and BC, then

(Pt. 8) First



$4(AB^2+CP^2) = 5AC^2 = 20PQ^2$

(3) If medians from B and C cut each other at 90°, then



$AB^2+AC^2 = 5BC^2$

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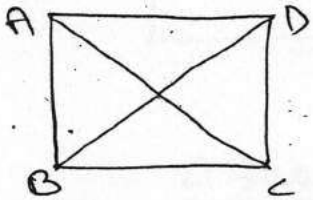
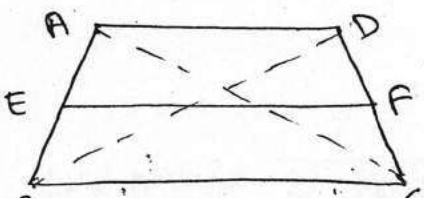
(4) Perimeter of a quadrilateral will be greater than sum of its diagonals



$AB+BC+CD+AD > BD+AC$

and $BD+AC > BC+CD$
$> CD+DA$
$> DA+AB$
$> AB+BC$

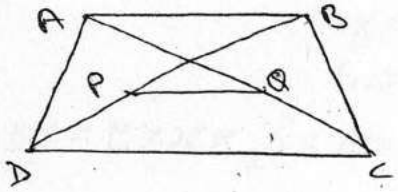
(5) ABCD = Trapezium ; AD ∥ BC, E & F are mid pts then,



~~$EF = \frac{1}{2}(BC+AD)$~~

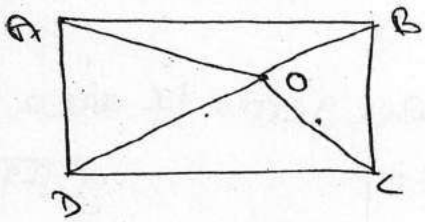
$EF = \frac{1}{2}(AB+CD)$

(6) P & Q are mid pts of Diagonals BD and AC, then



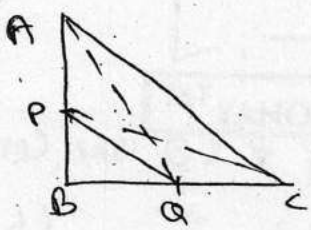
$$PQ = \frac{DC - AB}{2}$$

* (7) ABCD = Rectangle and O is any pt. inside it.



$$OA^2 + OC^2 = OB^2 + OD^2$$

* (8) $\angle B = 90°$, P & Q are pts. anywhere on AB and BC.



$$AQ^2 + CP^2 = AC^2 + PQ^2$$

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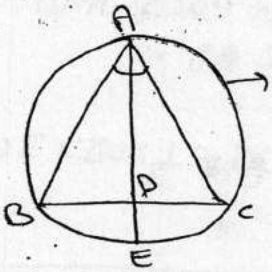
* (9) AD = Angle bisector of $\angle A$



$$AB \cdot AC + DE \cdot AE = AE^2$$

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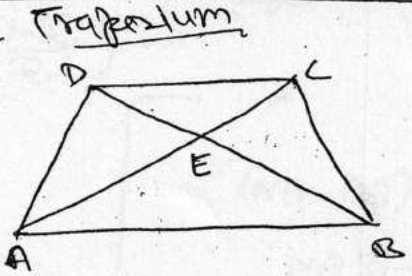
* (10) Trapezium



$$\frac{DE}{EB} = \frac{EC}{EA}$$

$$DE \times EA = EB \times EC$$

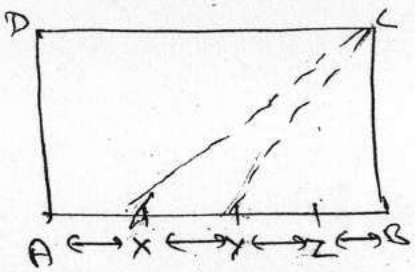
Diagonals of a trapezium divide each other in equal proportion

$\triangle AEB \sim \triangle DEC$

Similar triangles.

$$AC^2 + BD^2 = BC^2 + AD^2 + 2AB \cdot CD$$

Ther... $\frac{\text{Area}(\triangle xyc)}{\text{Area}(\square ABCD)} =$? (y, z. $\frac{1}{8}$)



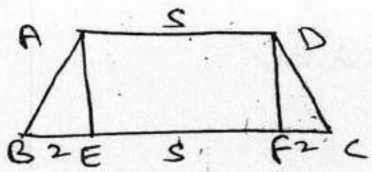
$xy = x$, $AB = 4x$, $BC = y$

Area ($\square ABCD$) $= 4x \cdot y$ cm^2

Area ($\triangle xyc$) $= \frac{1}{2} \times \text{Base} \times \text{Ht} = \frac{1}{2} \times x \times y = \frac{xy}{2}$

$\frac{\frac{xy}{2}}{4xy} = \frac{xy}{8xy} = \frac{1}{8}$

(20) ABCD = trapezium, AB = CD and AD = 5 cm, BC = 9 cm. If area of ABCD = 35 cm^2, then find length of CD = ? ($\sqrt{29}$ cm)



$\frac{1}{2}(5+9) \times DF = 35 \rightarrow DF = 5$ cm

$CD = \sqrt{DF^2 + FC^2} = \sqrt{25+4} = \sqrt{29}$

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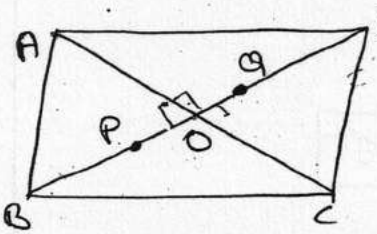
(21) ABCD = ~~trapezium~~ parallelogram, Diagonal BD = 18 cm. if P & Q are centroids of $\triangle ABC$ and $\triangle ADC$, then find length PQ = ? (6 cm)



Diagonals of a ||gm bisect each other.

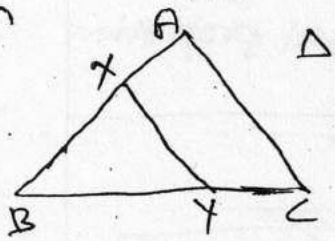
i.e. O is mid pt of AC & BD

OB = OD = 9 cm

$OP = \frac{1}{3} \times OB = 3$ cm, $OQ = \frac{1}{3} \times OD = 3$ cm

PQ = 3 + 3 = 6 cm

(22) XY || AC and xy divides $\triangle ABC$ into two parts equal in area. Find $\frac{AX}{AB}$ = ? ($\frac{\sqrt{2}-1}{\sqrt{2}}$)



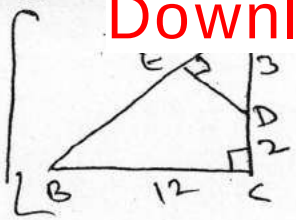
$\triangle ABC \sim \triangle BXY$

$\left(\frac{BA}{BX}\right)^2 = \frac{2}{1} \rightarrow BA = \sqrt{2}\,BX = \sqrt{2}(BA - AX)$

$\Rightarrow BA - \sqrt{2}\,BA = -\sqrt{2}\,AX$

$\text{Or } \sqrt{2}\,AX = \sqrt{2}\,BA(\sqrt{2}-1)$

$\frac{AX}{\quad} = \frac{\sqrt{2}-1}{\quad}$

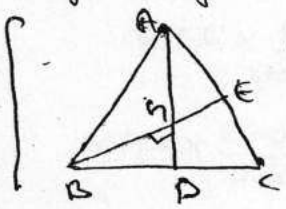


$\sqrt{13}$

So $\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$ so $\frac{13}{3} = \frac{5}{AE}$ so $AE = \frac{15}{13}$

---

(24) if medians AD and BE of any ΔABC intersect at G making right angle. if AD = 9, BE = 6 then BD = ? (5 cm)



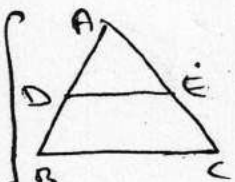
$GD = \frac{1}{3} \times 9 = 3$

$BG = \frac{2}{3} \times 6 = 4$

$BD = \sqrt{GD^2 + BG^2} = \sqrt{25} = 5$ cm

---

(25) In ΔABC, D & E are pts on AB such that DE || BC and DE divides the Δ into two equal areas, then $\frac{AD}{BD} = ?$ $(1 : \sqrt{2}-1)$



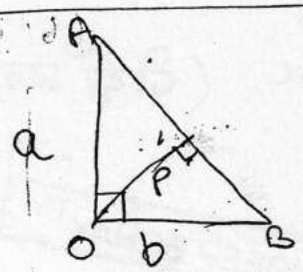
$\frac{AD^2}{AB^2} = \frac{1}{2} \rightarrow AB = \sqrt{2}\,AD$

$BD = AB - AD = \sqrt{2}\,AD - AD = AD(\sqrt{2}-1)$

$\frac{AD}{BD} = \frac{AD}{AD(\sqrt{2}-1)} = \boxed{\frac{1}{\sqrt{2}-1}}$

---

(26)



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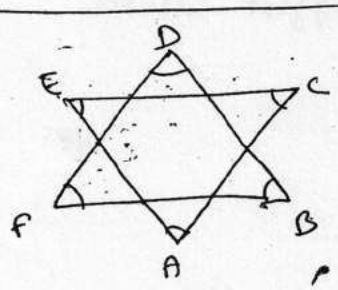
$\frac{1}{P^2} = \frac{1}{a^2} + \frac{1}{b^2}$ *

$\frac{1}{2} \times AB \times P = \frac{1}{2} a \times b \rightarrow \frac{a^2b^2}{P^2} = a^2 + b^2$

$\frac{1}{P^2} = \frac{1}{a^2} + \frac{1}{b^2}$

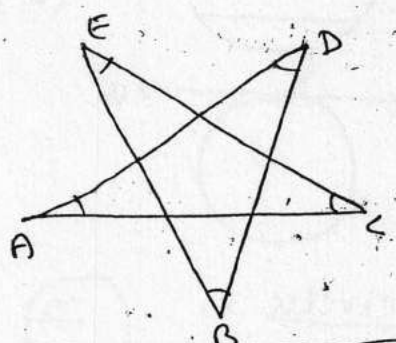
---

*



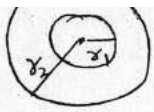
$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$

*

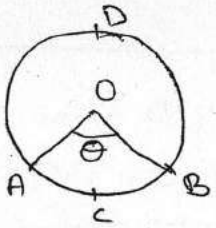


$\angle A + \angle B + \angle C + \angle D + \angle E = 180°$

* Concentric circles
(संकेन्द्रीय)



* Arc of a circle
(चाप)



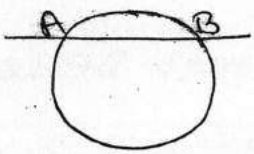
$\overset{\frown}{ACB}$ = minor Arc

$\overset{\frown}{ADB}$ = major Arc

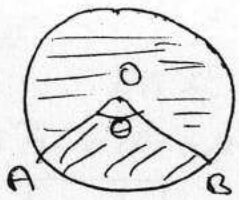
length of $\overset{\frown}{ACB} = \frac{\theta}{360} \times 2\pi r$

length of $\overset{\frown}{ADB} = \frac{360-\theta}{360} \times 2\pi r$

* Secant



* Sectors of a circle
(segments)



Minor segment → segment containing minor Arc (लघु खण्ड)

Area of minor segment = $\frac{\theta}{360} \times \pi r^2$

Major segment → segment containing major Arc (दीर्घ खण्ड)

Area of major segment = $\frac{360-\theta}{360} \times \pi r^2$

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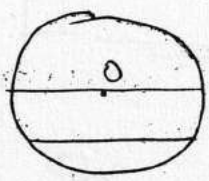
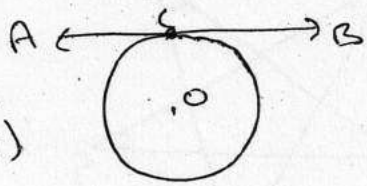
* chords
(जीवा)

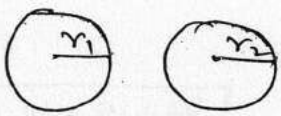


Diameter is the biggest chord.
(व्यास)

* Tangent
(स्पर्श रेखा)



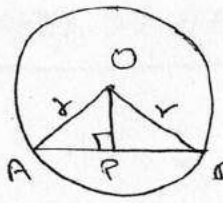
* Congruent circles → (सर्वांगसम)



if $r_1 = r_2$ then circles are Congruent.

* Imp

→ The ⊥r from the centre of a circle bisects the chord.

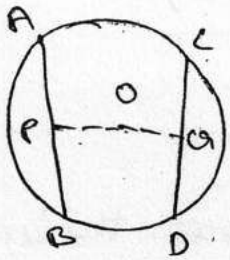


if OP ⊥r AB then AP = PB

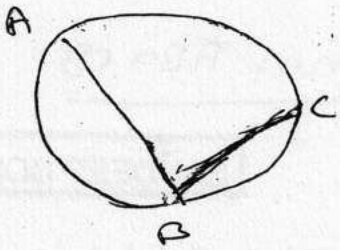
and conversely, the line joining the centre to the mid pt. of a chord is ⊥r to the chord.

if AP = PB then OP ⊥r AB

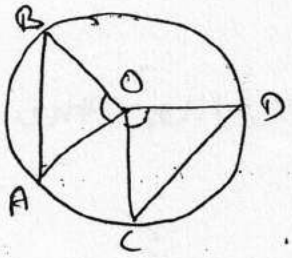
→ The ⊥r bisectors of two chords of a circle intersects at centre



→ There is one & only one circle passing through three non-collinear pts.



→ equal chords subtend equal angle at the centre and conversely if angles subtended by chords at the centre are equal, then the chords are equal.

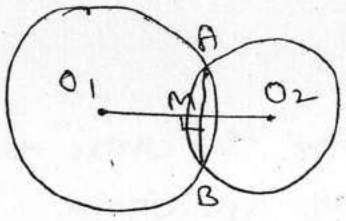


if AB = CD then ∠AOB = ∠COD

if ∠AOB = ∠COD then AB = CD

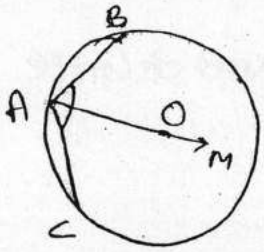
→ Same rule applied for Arcs (चाप)

the line joining their centres is the ⊥r bisector of their common chord.



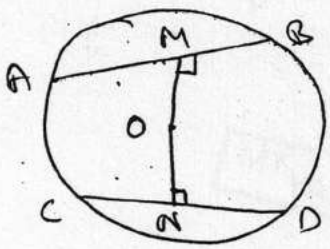
O_1O_2 is ⊥r bisector of AB i.e. AM = MB

→ if two chords AB & AC are equal then the centre of circle lies on the angle bisector of ∠BAC.



Centre O lies on AM (⊥r bisector of ∠BAC)

→ Equal chords of a circle are equidistant from the centre and ~~con~~ vice-versa.



if AB = CD then OM = ON

if OM = ON then AB = CD

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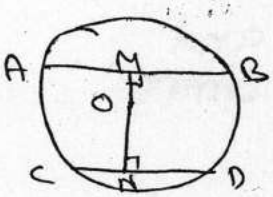
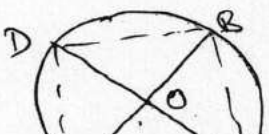
→ Larger chord is nearer to centre and vice-versa.



If AB > CD then OM < ON

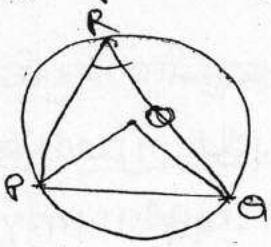
so Diameter is the biggest chord.

→ if two chords of a circle bisect each other then they must be diameters.

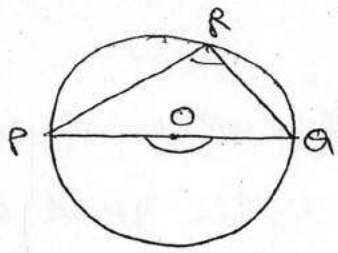


AB and CD are diameters

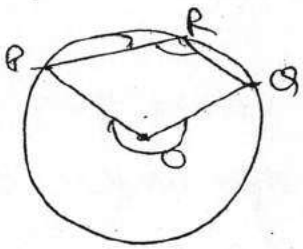
* The ... is double ... subtended by it at any pt. on the rem. part of circle.



minor arc



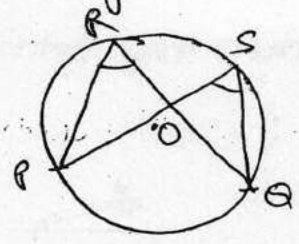
Semicircle

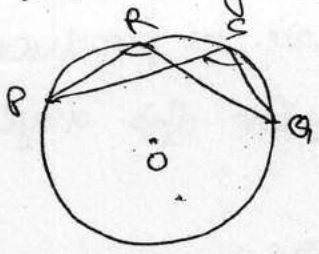


Major Arc

In all cases, $\angle POQ = 2\angle PRQ$

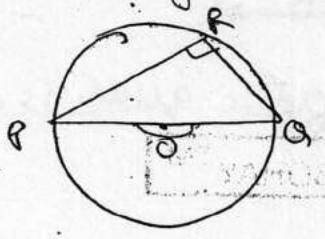
* Angles in the same segment of a circle are equal.





$\angle PRQ = \angle PSQ$

* The angle in a semicircle is a right angle.



$\angle PRQ = 90°$

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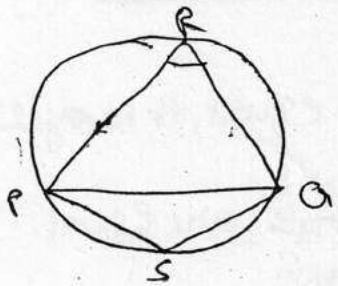
$\left[\angle PRQ = \frac{1}{2}\angle POQ = \frac{180}{2} = 90°\right]$

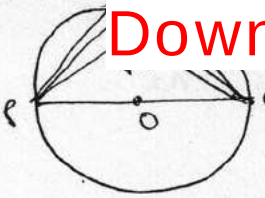
* Any angle subtended by a minor arc in the alternate segment is acute and any angle subtended by a major arc in the alternate segment is obtuse.



$\angle PRQ < 90°$

$\angle PSQ > 90°$

* The Diameter of a circle subtends ... at a point in the interior of circle.

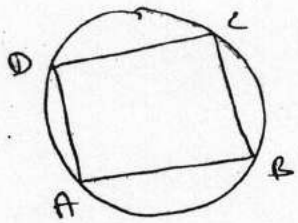


$\angle PSQ = 90°$

$\angle PTQ < 90°$

(चक्रीय चतुर्भुज)

* The sum of either pair of opp. angles of a cyclic quadrilateral is 180° or opp. angles of a cyclic quad. are supplementary.

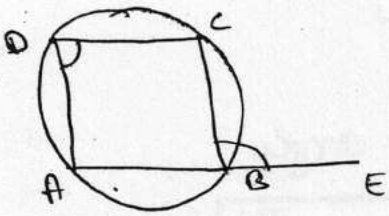


$\angle A + \angle C = 180°$

$\angle B + \angle D = 180°$

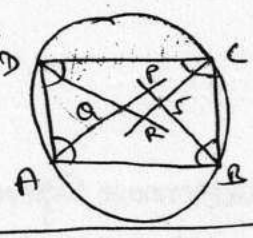
& vice-versa

* if a side of a cyclic quad. is produced then the exterior angle is equal to the interior opp. angle.



$\angle CBE = \angle ADC$

* The quad. formed by the angle bisectors of a cyclic quad. is also cyclic.



PQRS is also cyclic

---

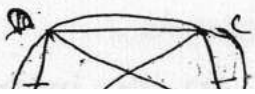
* if two sides of a cyclic quad. are parallel then the rem. two sides are equal and diagonals are also equal.

or

if two non parallel sides of a trapezium are equal, it is cyclic

or

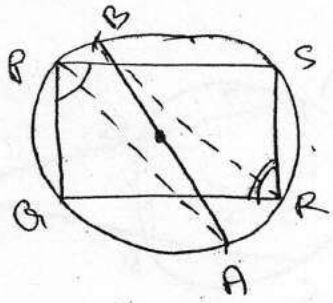
A cyclic trapezium is isosceles and its diagonals are equal.



if AB || CD
then AD = BC

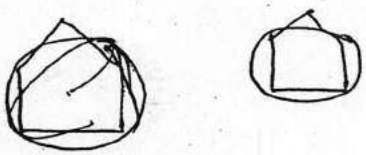
प्रमाण

* if PQRS intersect the circle at A & B resp, then AB = diameter.

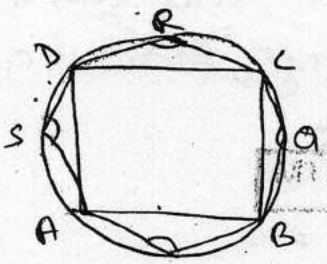


AB = Diameter.

* Any four vertices of a regular pentagon are concyclic

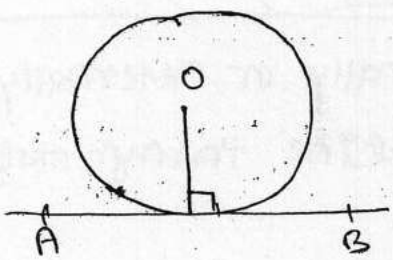


* The sum of the angles in the four segments exterior to a cyclic quad. is equal to 6 right angles (540°)



$\angle P + \angle Q + \angle R + \angle S = 6 \times 90 = 540$

* A tangent to a circle is ⊥r to the radius through the point of contact.



OP ⊥r AB

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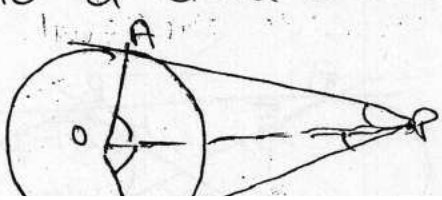
* The length of two tangents drawn from an external pt. to a circle are equal

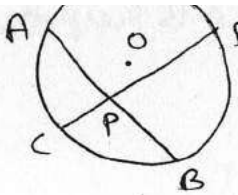


PA = PB

and $\angle POA = \angle POB$ → equal angles at centre

* if

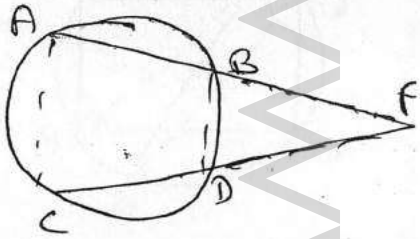
1) inside the circle



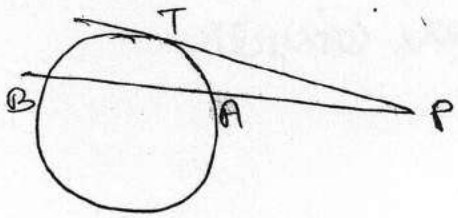
$PA \times PB = PC \times PD$

2) outside the circle

$PA \times PB = PC \times PD$



* if PAB is a secant & PT is a tangent



$PT^2 = PA \times PB$

* If a chord is drawn through the point of contact of a tangent to a circle, then the angle which this chord makes with the tangent is equal to the angle formed in the alternate segment.

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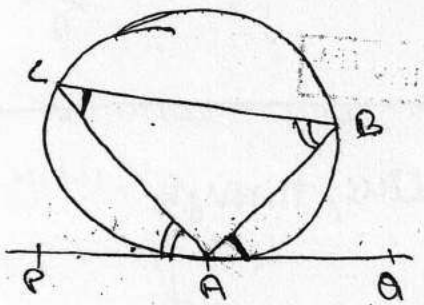

$\angle BAQ = \angle BCA$

$\angle PAC = \angle CBA$

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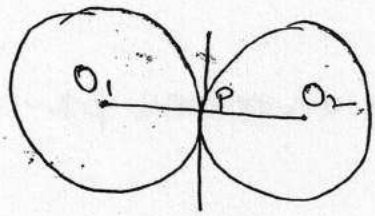
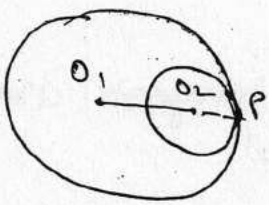
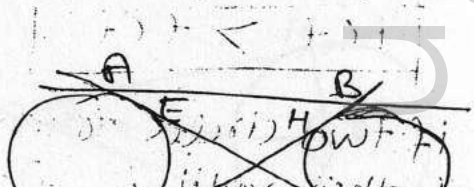
* if two circles touch each other internally or externally then the point of contact lies on the line through the centres



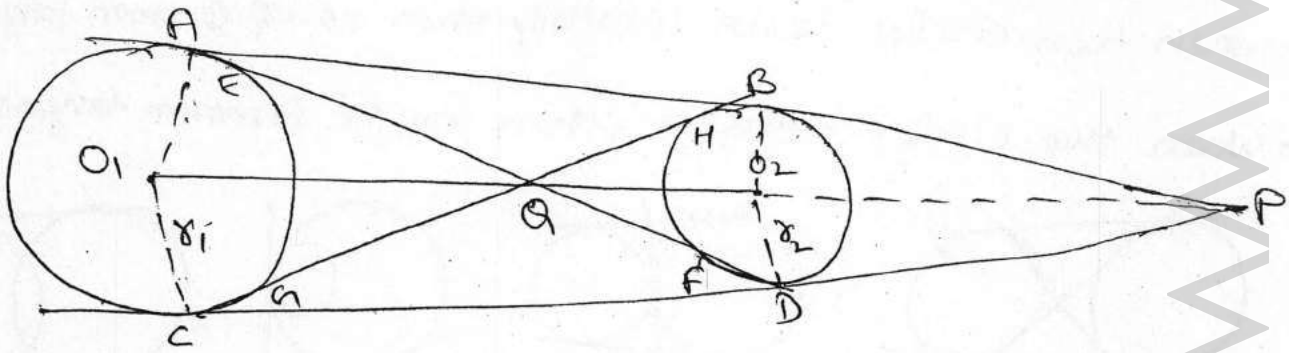


O_1, O_2 & P are collinear

* Direct common tangents → AB & CD



* The transverse common tangents to two circles divide the line segment joining the two centres in the ratio of their radii.



AB, CD = Direct common tangents → divide externally

EF, GH = Transverse common tangents → divide internally

→ P divides O_1O_2 externally in the ratio $r_1 : r_2$

$$\frac{O_1P}{O_2P} = \frac{r_1}{r_2}$$

→ Q divides O_1O_2 internally in the ratio $r_1 : r_2$

$$\frac{O_1Q}{O_2Q} = \frac{r_1}{r_2}$$

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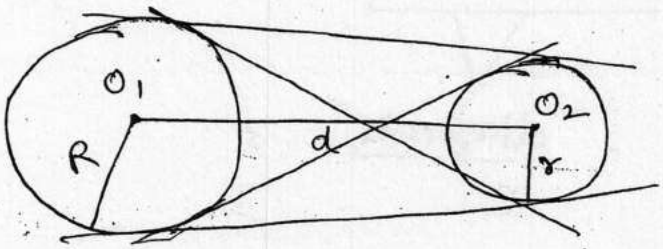
*



R → Radius of bigger circle

r → Radius of smaller circle

Length of Direct common tangent (dct) = $\sqrt{d^2 - (R-r)^2}$ → $(R-r)^2$

Length of Transverse common tangent (tct) = $\sqrt{d^2 - (R+r)^2}$ → $(R+r)^2$

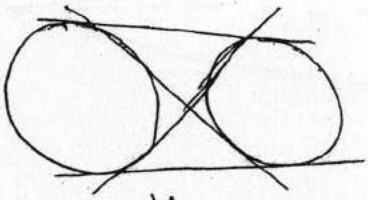
So $dct > tct$

* If two circles touch externally, then diff. b/w their centres is sum of their radii & if they touch internally then diff. b/w their

→ When two circles touch externally then no. of common tangents = 3

→ When two circles touch internally then no. of common tangents = 1

→ When two circles intersect, then no. of common tangents = 2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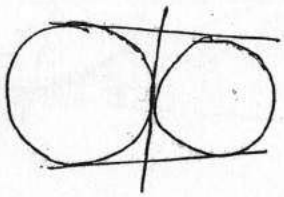
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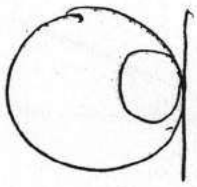
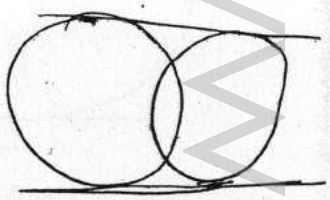

1


2

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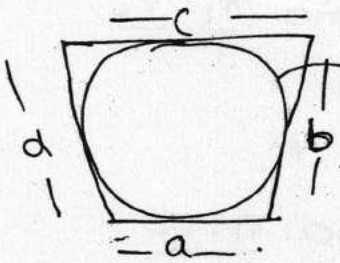
*

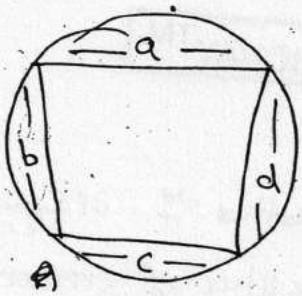


→ Area of circle = $\sqrt{abcd}$

and $\boxed{a+c = b+d}$

---

*



cyclic quad.

$\boxed{ac+bd = d_1 \times d_2}$

diagonals.

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*

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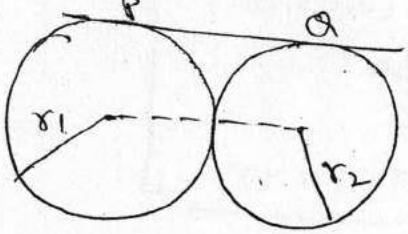
externally. and PQ is common tangent. Find PQ^2 = ? $(4r_1r_2)$



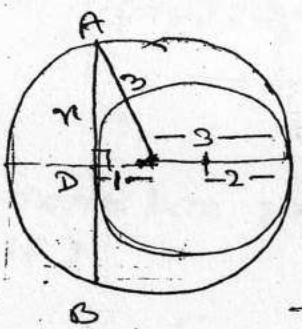
Direct tangent $PQ = \sqrt{(r_1+r_2)^2-(r_1-r_2)^2}$

$PQ^2 = r_1^2+r_2^2+2r_1r_2-(r_1^2+r_2^2-2r_1r_2)$

$= r_1^2+r_2^2+2r_1r_2-r_1^2-r_2^2+2r_1r_2$

$PQ^2 = 4r_1r_2$

(28) Two circles touch each other internally. Their radii are 2 cm & 3 cm. What will be length of longest chord of bigger circle outside the smaller circle? $(4\sqrt{2}$ cm)



$AD = \sqrt{3^2-1^2} = \sqrt{8} = 2\sqrt{2}$

$AB = 2 \times 2\sqrt{2} = 4\sqrt{2}$

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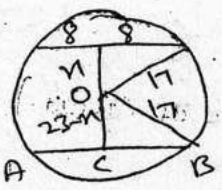
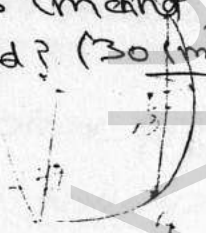
(29) In a circle of radius ~~[illegible]~~ 17 cm two ll chords are present on the opp. sides of diameter. Distance b/w them is 23 cm and length of one chord is 16 cm. Find length of other chord? (30 cm)



$x = 15$

$OC = 23-15 = 8$

$BC = 15$ so $AB = 2 \times 15 = 30$

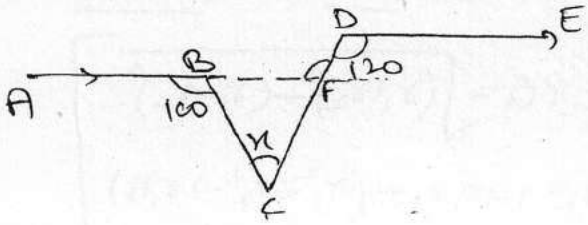


(30) If radii of two circles be 6 cm and 3 cm and length of TCT be 8 cm then find distance b/w their centres? $(\sqrt{145}$ cm)

$8 = \sqrt{d^2-(6+3)^2} \rightarrow d = \sqrt{145}$

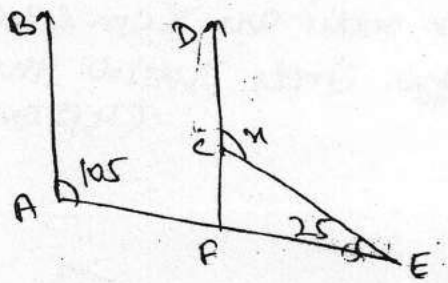
(31) If a chord of a circle of radius 5 cm is the tangent to other circle of radius 3 cm, both the circles being concentric, then find the length of chord. ? (8 cm)

(32) ABC (40)



$\angle BFD = \angle EDC = 90°$ (alternate)
$\angle CFB = 180 - 120 = 60°$
$\angle FBC = 180 - 100 = 80°$
$\angle BCD = 180 - (80+60) = 40°$

(33) AB || CD, $\angle BAE = 105$, $\angle AEC = 25$, $\angle DCE = ?$ (130)



$\angle CFE = \angle BAF = 105$ (corresponding)
$x = \angle CFE + \angle FEC = 25 + 105 = 130°$
(external angle = opp. two angles sum)

(34) What is the ratio b/w the sum of internal angles and sum of external angles of a octagone? (3:1)

sum of ext. angles = 360
sum of int. angles = $(8-2) \times 180 = 1080 \rightarrow 1080 : 360 \rightarrow 3:1$

(35) If ratio of int. angles of a pentagon are 1:2:3:5:9 then the smallest angle is? (27°)

$20x = (5-2) \times 180 \rightarrow 540$
$x = 540/20 = 27°$

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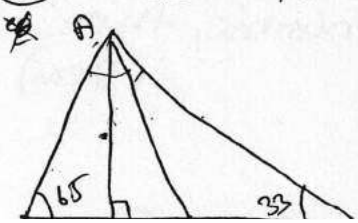
(36) if no. of diagonals in a polygon are 27 then find no. of sides? (9)

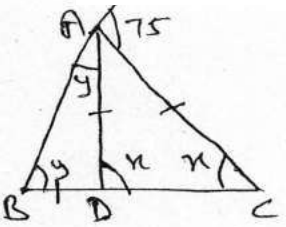
$\frac{n(n-3)}{2} = 27 \rightarrow n^2 - 3n - 54 = 0 \rightarrow (n-9)(n+6) = 0$, $n = 9$

(37) AM ⊥ BC, AN is angle bisector of $\angle A$ and $\angle B = 65°$, $\angle C = 33°$, Find $\angle MAN$? (16)



$\angle A = 180 - (65+33) = 82°$
$\angle NAC = 41°$, $\angle CNA = 180 - (33+41) = 106°$
$\angle ANM = 180 - 106 = 74°$, $\angle MAN = 180 - (74+90) = 16°$

(50)

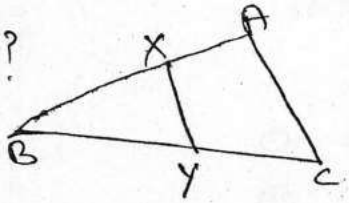


$\angle ADC = y+y = 2y$

$x = 2y \rightarrow y = \frac{x}{2}$

$\frac{x}{2} + 180 - 2x + 75 = 180 \rightarrow x = 50°$

(39) XY ∥ AC and it divides Δ into two equal parts then $\frac{AX}{AB} = ?$

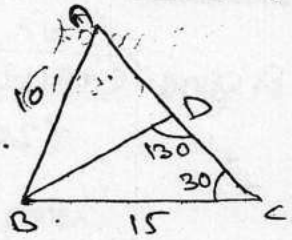


$\text{Area}(\Delta ABC) = 2 \times \text{Area}(\Delta BXY)$

$\frac{BX^2}{AB^2} = \frac{\text{Area}(\Delta BXY)}{\text{Area}(\Delta ABC)} = \frac{1}{2} \rightarrow \frac{AM}{AB} = \frac{1}{\sqrt{2}}$ $\left(\frac{1}{\sqrt{2}}\right)$

(40) AD : DC = 2 : 3, AB = 10 cm, BC = 15 cm, $\angle BDC = 130°$, $\angle BCD = 30°$.

Find $\angle ABC = ?$ (40°)



$\frac{AD}{DC} = \frac{2}{3}$, $\frac{AB}{BC} = \frac{10}{15} = \frac{2}{3}$, $\frac{BA}{BC} = \frac{CD}{DA} = \frac{2}{3}$

so BD is angle bisector of $\angle B$

$\angle ABC = 2 \angle CBD = 2 \times 20 = 40°$

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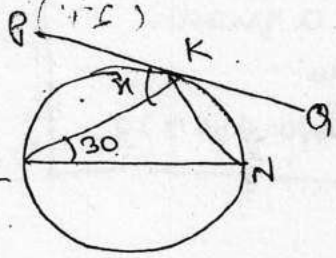
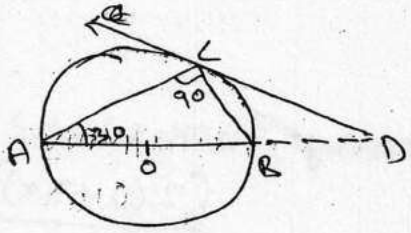
(41) PQ is tangent at K, LN = Diameter, if $\angle KLN = 30°$, then $\angle PKL = ?$



$\angle LNK = 180 - (90+30) = 60°$

$\angle PKL = \angle LNK = 60°$ (alternate segment angle) (60)

(42) AB = Diameter, AC = chord, $\angle BAC = 30°$, Find $\angle BDC = ?$ (30°)



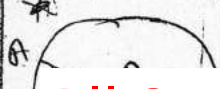
$\angle ABC = 180 - (90+30) = 60°$

$\angle CBD = 180 - 60 = 120$

$\angle BCD = 30°$ (alternate segment angle b/w chord & tangent)

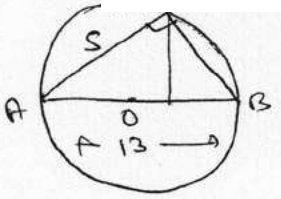
$\angle BDC = 180 - (120+30) = 30°$

(43) AB = Diameter with centre O. OE = EB, CE = 6 cm, ED = 2 cm, And $x = ?$



$OE + EB = x$, $OE = EB = x/2$ (4 cm)

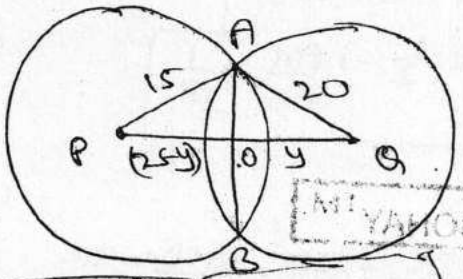
(30 cm²)



$BC = \sqrt{13^2 - 5^2} = 12$

Area $= \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$

(25) Radii of two circles are 15 cm & 20 cm and their centres are 25 cm apart. Find length of common chord? (24 cm)



Law of triplet (3,4,5) ×3, ×4

PQ = 25 cm

$AO^2 = 15^2 - (25-y)^2$ — ①

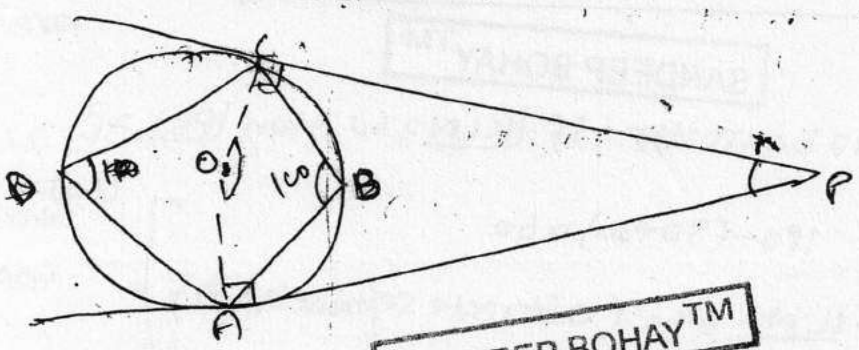
$AO^2 = 20^2 - y^2$ — ②

① = ② → $225 - 625 - y^2 + 50y = 400 - y^2 \Rightarrow y = 16$

$AO^2 = 20^2 - 16^2 = 144 \rightarrow AO = 12$

$AB = 2 \times 12 = 24$ cm

(26) ABCD is a cyclic quad. Tangents drawn at A and C meet at P. if $\angle ABC = 100°$, find $\angle APC$? (20)



$\angle D = 180 - 100 = 80°$

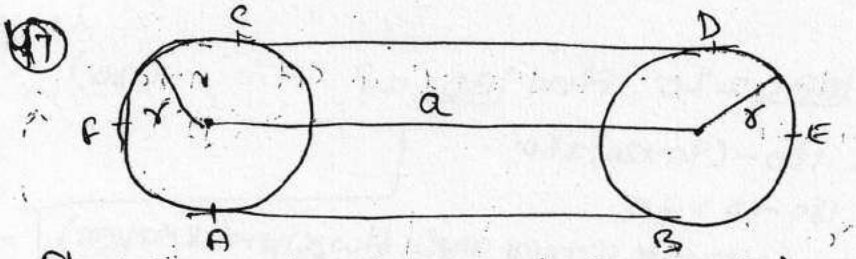
$\angle AOC = 2 \times 80 = 160$

AOCP is a quad.

$\angle A = \angle C = 90°$

$\angle APC = 180 - 160 = 20°$

(27)

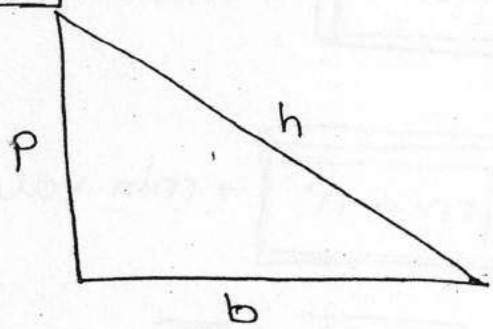


Find the min. length of Belt which passes through both wheels?

$AB + BED + CD + CFA = a + \frac{1}{2}(2\pi r) + a + \frac{1}{2}(2\pi r)$

$= 2(a + \pi r)$

BASIC :→



sin cos tan
P b P ⇒ Pandit Bhadri Parsad
h h b Har Har Bole
cosec sec cot

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→ $-1 \le \sin\theta \le +1$ ($\sin\theta = 1$ at $90°$ and -1 at $270°$)

→ $-1 \le \cos\theta \le +1$ ($\cos\theta = 1$ at $0°$ and -1 at $180°$)

→ $\sec\theta \ge 1 \quad \le -1$

→ $\text{cosec}\,\theta \ge 1 \quad \le -1$

$\sin 0°$	$\sin 90°$	$180°$	$270°$	$360°$
0	1	0	−1	0

$\cos 0°$	$\cos 90°$	$\cos 180°$	$270°$	$360°$
1	0	−1	0	1

$-\infty \le \tan\theta \le \infty$

$-\infty \le \cot\theta \le \infty$

$\sin\theta \cdot \text{cosec}\,\theta = 1$

$\cos\theta \cdot \sec\theta = 1$

$\tan\theta \cdot \cot\theta = 1$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\text{cosec}^2\theta - \cot^2\theta = 1$

$m\sin\theta \pm n\cos\theta$

or $m\sin\theta \pm n\sin\theta$

or $m\cos\theta \pm n\cos\theta$

is $\boxed{\sqrt{m^2+n^2}}$ → max. value

and Minimum value is $\boxed{-\sqrt{m^2+n^2}}$ → min. value

eg → Max. value of $4\sin\theta + 5\cos\theta = \sqrt{4^2+5^2} = \sqrt{41}$

min. value = $-\sqrt{41}$

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T2

$\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \boxed{\tan 45^\circ} \dots \tan 89^\circ = 1$

$\cot 1^\circ \cot 2^\circ \dots \boxed{\cot 45^\circ} \dots \cot 89^\circ = 1$

T3

$\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \underbrace{\cos 90^\circ}_{(0)} = 0$

$\cos 1^\circ \cos 2^\circ \dots \underbrace{\cos 90^\circ}_{(0)}$ to (cos 90° से बड़ा) $= 0$

T4

$\sin 1^\circ \sin 2^\circ \sin 3^\circ \dots \underbrace{\sin 180^\circ}_{(0)} = 0$

$\sin 1^\circ \sin 2^\circ \dots \underbrace{\sin 180^\circ}_{(0)} \dots \sin > 180 = 0$

Q① Find min. value of $\sin\theta + \cos\theta$?

$\left[-\sqrt{1^2+1^2} = -\sqrt{2}\right]$

Q② $\sin 1^\circ \sin 2^\circ \sin 3^\circ \dots \sin 179^\circ$?

→ $x = 30$

Q(10) if $x + \frac{1}{x} = 2\cos\theta$ then $x^3 + \frac{1}{x^3} = ?$ $(2\cos 3\theta)$

→ $\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3 \times \frac{1}{x}\left(x+\frac{1}{x}\right)$

$x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$= (2\cos\theta)^3 - 3 \times 2\cos\theta \Rightarrow 8\cos^3\theta - 6\cos\theta$

$= 2(4\cos^3\theta - 3\cos\theta) \Rightarrow \boxed{2\cos 3\theta}$

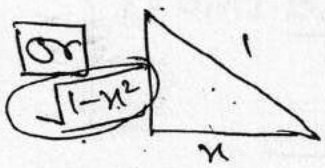
$\boxed{4\cos^3\theta - 3\cos\theta = \cos 3\theta}$

Q(11) if $\cos\theta = x$ then $\tan\theta = ?$ $\left(\frac{\sqrt{1-x^2}}{x}\right)$

→ $\sec^2\theta - \tan^2\theta = 1 \Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1}$

$= \sqrt{\frac{1}{\cos^2\theta} - 1} \Rightarrow \sqrt{\frac{1}{x^2} - 1} \Rightarrow \frac{\sqrt{1-x^2}}{x}$

Or



$\boxed{\tan\theta = \frac{\sqrt{1-x^2}}{x}}$

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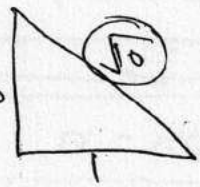
Q(12) if $0 < \theta < 90$ and $\tan\theta = \sqrt{6+\sqrt{6+\sqrt{6+\cdots\infty}}}$ then $\sec^2\theta = ?$

→ $\tan\theta = 3 \Rightarrow \sec\theta$



$\sec^2\theta = \frac{\sqrt{10}}{1}$

$\boxed{\sec^2\theta = 10}$

Q(13) if $\alpha + \beta = 90°$ & $\alpha = 2\beta$ then $\cos^2\alpha + \sin^2\beta = ?$

→ $\alpha = 60°, \beta = 30° \Rightarrow \cos^2 60° + \sin^2 30°$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$

m^2+n^2 ?

→ $a^2\cos^2\theta + b\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$

$= m^2+n^2$

→ $a^2(\cos^2\theta+\sin^2\theta) + b^2(\sin^2\theta+\cos^2\theta) = m^2+n^2$

→ $\boxed{a^2+b^2 = m^2+n^2}$

Q(13) if $\sin^2\alpha = \cos^3\alpha$ then find $\cot^6\alpha - \cot^2\alpha = ?$ (1)

$\frac{\sin^2\alpha}{\sin^3\alpha} = \frac{\cos^3\alpha}{\sin^3\alpha} \rightarrow \cot^3\alpha = \operatorname{cosec}\alpha$, $\cot^6\alpha = \operatorname{cosec}^2\alpha$

so $\cot^6\alpha - \cot^2\alpha \rightarrow \underline{\operatorname{cosec}^2\alpha - \cot^2\alpha = 1}$

Q(12) if $A+B+C = 90$ and $\cot A + \cot B + \cot C = 36$, $\cot A \cdot \cot B = 12$ then $\cot C$? (3)

when $A+B+C=90$ then $\boxed{\cot A + \cot B + \cot C = \cot A \cdot \cot B \cdot \cot C}$

so $36 = 12 \cdot \cot C \rightarrow \underline{\cot C = 3}$

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Q(11) if $9\cos\theta + 12\sin\theta = 15$ then find $\cot\theta = ?$ $(\frac{3}{4})$

$\frac{9}{15}\cos\theta + \frac{12}{15}\sin\theta = 1$ and we know $\cos\theta\cdot\cos\theta + \sin\theta\cdot\sin\theta = 1$

so $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$ so $\cot\theta = \frac{3}{4}$

Or if $a\sin\theta + b\cos\theta = c$

then a = perpendicular, b = base, c → hypotenuse

Or $ax+by=m$, $ay-bx=n$

then $\boxed{(a^2+b^2)(x^2+y^2) = m^2+n^2}$

where $x, y \rightarrow \sin\theta, \cos\theta, \tan\theta$ etc.

→ Q(14) Shortcut

Q③

$$\rightarrow \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \times \sqrt{\frac{1-\sin\theta}{1-\sin\theta}} \Rightarrow \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} \Rightarrow \sqrt{\left(\frac{1-\sin\theta}{\cos\theta}\right)^2} \Rightarrow \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \Rightarrow \boxed{\sec\theta - \tan\theta}$$

Q④

T5 ⇔

if $\sec\theta \pm \tan\theta = x$ then $\sec\theta = \frac{x^2+1}{2x}$

Q④ if $\sec\theta + \tan\theta = 3$ then find $\sec\theta$? (5/3)

$$\rightarrow \sec\theta = \frac{x^2+1}{2x} = \frac{9+1}{6} = \frac{10}{6} = \boxed{\frac{5}{3}}$$

Q⑤ if $\sec\theta + \tan\theta = 2$ then find $\tan\theta$? (3/4)

$$\rightarrow \sec\theta = \frac{x^2+1}{2x} = \frac{2^2+1}{2\times 2} = \frac{5}{4}$$

~~$\tan^2\theta = \sec^2\theta - 1 \Rightarrow \frac{25}{16} - 1 = \frac{9}{16}$~~ $\tan\theta = 2 - \frac{5}{4} = \frac{3}{4}$

$\boxed{\tan\theta = \frac{3}{4}}$

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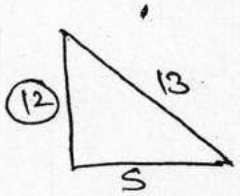
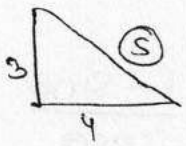
Q⑥ $\cos\theta = \frac{5}{13}$ then find $\tan^2\theta + \sin^2\theta = ?$



$$\tan^2\theta + \sin^2\theta = \left(\frac{12}{5}\right)^2 + \left(\frac{12}{13}\right)^2$$

$$= \frac{144 \times 194}{25 \times 169}$$

Q7) $3\sin\theta - 4\cos\theta$ (-25/7)



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\rightarrow \frac{3\times\frac{3}{5} + 4\times\frac{4}{5}}{3\times\frac{3}{5} - 4\times\frac{4}{5}} = -\frac{25}{7}$

OR $\frac{3\frac{\sin\theta}{\cos\theta} + 4}{3\frac{\sin\theta}{\cos\theta} - 4}$

OR $\tan\theta = \frac{\sin\theta}{\cos\theta}$ T6

$\frac{3}{4} = \frac{\sin\theta}{\cos\theta} = \frac{3\times3 + 4\times4}{3\times3 - 4\times4} = \frac{25}{7}$

⇒ T7

if $\sin\theta + \cos\theta = x$
then $\sin\theta - \cos\theta = \sqrt{2 - x^2}$

OR

if $\sin\theta - \cos\theta = x$
then $\sin\theta + \cos\theta = \sqrt{2 - x^2}$

Proof → $\sin\theta + \cos\theta = x$

squaring both sides → $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = x^2$

$2\sin\theta\cos\theta = x^2 - 1$

$(\sin\theta - \cos\theta)^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 - (x^2 - 1)$

$= 1 - x^2 + 1$

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$\sin\theta - \cos\theta = \sqrt{2 - x^2}$

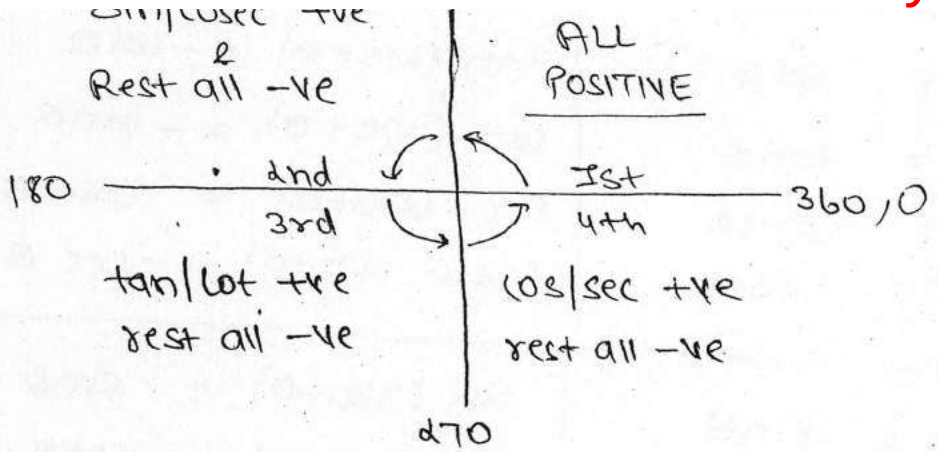
Q8) $\sin\theta + \cos\theta = \sqrt{2}$ then $\sin\theta - \cos\theta = ?$

→ $\sin\theta - \cos\theta = \sqrt{2 - (\sqrt{2})^2} = \sqrt{2-2} = 0$

Q9) $\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1, \theta = 45°$ To find θ.

Q9) $\frac{\sin x}{1+\cos x} + \frac{\sin x}{1-\cos x} = 4$ then find x ? (30)



$90° \pm \theta$, $270° \pm \theta$ → change

$180° \pm \theta$, $360° \pm \theta$ → No change.

sin → cos , cos → sin , tan → cot , cot → tan , sec → cosec , cosec → sec

From → 0 to 90°

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$\sin\theta$ → increase	$\cos\theta$ → decrease
$\tan\theta$ → increase	$\cot\theta$ → decrease
$\sec\theta$ → increase	$\operatorname{cosec}\theta$ → decrease

→ $\sin(90-\theta) = \cos\theta$
$\cos(90-\theta) = \sin\theta$
$\tan(90-\theta) = \cot\theta$
$\cot(90-\theta) = \tan\theta$
$\sec(90-\theta) = \operatorname{cosec}\theta$
$\operatorname{cosec}(90-\theta) = \sec\theta$

$\sin(90+\theta) = \cos\theta$
$\cos(90+\theta) = -\sin\theta$
$\tan(90+\theta) = -\cot\theta$
$\cot(90+\theta) = -\tan\theta$
$\sec(90+\theta) = -\operatorname{cosec}\theta$
$\operatorname{cosec}(90+\theta) = \sec\theta$

→ $\sin(180-\theta) = \sin\theta$
$\cos(180-\theta) = -\cos\theta$
$\tan(180-\theta) = -\tan\theta$
$\cot(180-\theta) = -\cot\theta$
$\sec(180-\theta) = -\sec\theta$

$\sin(180+\theta) = -\sin\theta$
$\cos(180+\theta) = -\cos\theta$
$\tan(180+\theta) = \tan\theta$
$\cot(180+\theta) = \cot\theta$
$\sec(180+\theta) = -\sec\theta$

$\tan(270-\theta) = \cot\theta$

$\cot(270-\theta) = \tan\theta$

$\sec(270-\theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(270-\theta) = -\sec\theta$

$\tan(270+\theta) = -\cot\theta$

$\cot(270+\theta) = -\tan\theta$

$\sec(270+\theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(270+\theta) = -\sec\theta$

→ $\sin(360-\theta) = -\sin\theta$

$\cos(360-\theta) = \cos\theta$

$\tan(360-\theta) = -\tan\theta$

$\cot(360-\theta) = -\cot\theta$

$\sec(360-\theta) = \sec\theta$

$\operatorname{cosec}(360-\theta) = -\operatorname{cosec}\theta$

$\sin(360+\theta) = \sin\theta$

$\cos(360+\theta) = \cos\theta$

$\tan(360+\theta) = \tan\theta$

$\cot(360+\theta) = \cot\theta$

$\sec(360+\theta) = \sec\theta$

$\operatorname{cosec}(360+\theta) = \operatorname{cosec}\theta$

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	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

→ $\tan(360+60) \Rightarrow \tan 60 = \sqrt{3}$

Q(16) $\sin 210°$?

→ $\sin(180+30) \Rightarrow -\sin 30° = -\frac{1}{2}$

Q(17) $\sec 315°$?

→ $\sec(360-45) = \sec 45° = \sqrt{2}$

Q(18) $\sin 2295°$?

→ $\sin(6\times360+135) \Rightarrow \sin(135°) \Rightarrow \sin(180-45)$

→ $\sin 45 = +\frac{1}{\sqrt{2}}$

Q(19) $\tan 1485°$?

→ $\tan(4\times360+45°) \Rightarrow \tan 45° = +1$,

Q(20) $\frac{\sin 72°}{\cos 18°}$?

→ $\frac{\sin(90-18)}{\cos 18°} \Rightarrow \frac{\cos 18°}{\cos 18°} = 1$

T8

if $\theta_1+\theta_2=90°$ then $\frac{\sin\theta_1}{\cos\theta_2}=1$ ✓

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\sin 2A = 2\sin A \cos A = \dfrac{2\tan A}{1+\tan^2 A}$

$\cos 2A = \dfrac{1 - 2\sin^2 A}{= \cos^2 A - \sin^2 A} = \dfrac{2\cos^2 A - 1}{} = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

Proof:- $\sin 2A = \sin(A+A) = \sin A \cos A + \sin A \cos A = 2\sin A \cos A$

$\Rightarrow 2\sin A \cos A \times \dfrac{\cos A}{\cos A} = \dfrac{2\sin A \cos^2 A}{\cos A} = \dfrac{2\tan A}{\sec^2 A} = \dfrac{2\tan A}{1+\tan^2 A}$

$\cos 2A = \cos(A+A) = \cos A \cos A - \sin A \sin A = \cos^2 A - \sin^2 A$

$\Rightarrow (\cos^2 A - (1 - \cos^2 A) \Rightarrow \cos^2 A - 1 + \cos^2 A = 2\cos^2 A - 1$

$\Rightarrow 2(1 - \sin^2 A) - 1 \Rightarrow 2 - 2\sin^2 A - 1 \Rightarrow 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

$$\tan 3A = \frac{3\tan A - 3\tan^3 A}{1 + 3\tan^2 A}$$

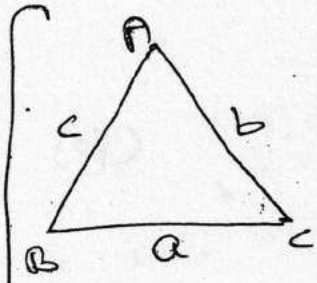
$$\sin C + \sin D = 2\sin\frac{C+D}{2} \cdot \cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\sin\frac{C-D}{2} \cdot \cos\frac{C+D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2} \cdot \cos\frac{C-D}{2}$$

$$\cos C - \cos D = 2\sin\frac{C+D}{2} \cdot \sin\frac{D-C}{2}$$

hit & trial use



$$\left[\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\right] = 2R$$ → circumradius

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$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Cosine Rule

$$\sin\theta \cdot \sin 2\theta \cdot \sin 4\theta = \frac{1}{4}\sin 3\theta$$

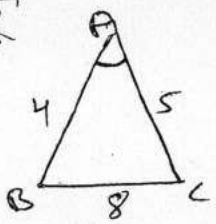
$$\cos\theta \cdot \cos 2\theta \cdot \cos 4\theta = \frac{1}{4}\cos 3\theta$$

→ $4\theta < 90°$

$$\tan\theta \cdot \tan 2\theta \cdot \tan 4\theta = \tan 3\theta$$ → $4\theta \leq 90°$

→ $\sin 90^\circ = 1$

(92) In $\triangle ABC$, $AB = 4$ cm, $BC = 8$ cm, $AC = 5$ cm then $\cos A$?



$$\cos A = \frac{4^2+5^2-8^2}{2\times4\times5} = \frac{16+25-64}{40} = \frac{-23}{40}$$

(93) $\cos 10^\circ \cos 20^\circ \cos 40^\circ = ?$ $(\sqrt{3}/8)$

→ $\frac{1}{4}\cos 3\theta = \frac{1}{4}\times\cos 30^\circ = \frac{1}{4}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$

(94) $\sin 10^\circ \times \sin 20^\circ \times \sin 40^\circ \times \sin 60^\circ = ?$ $(\sqrt{3}/16)$

$\frac{1}{4}\sin 3\theta \times \sin 60^\circ \rightarrow \frac{1}{4}\times\frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{16}$

(95) $\sin 10^\circ \times \sin 50^\circ \times \sin 70^\circ = ?$ $(1/8)$

→ $\sin(90-80)\times\sin(90-40)\times\sin(90-20)$

$\cos 80^\circ \cdot \cos 40^\circ \cdot \cos 20^\circ = \frac{1}{4}\cos 30^\circ = \frac{1}{4}\times\cos 60^\circ = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$

(96) $\tan\frac{\pi}{36}\cdot\tan\frac{\pi}{18}\cdot\tan\frac{\pi}{9} = ?$ $(2-\sqrt{3})$

→ $\tan 5^\circ \cdot \tan 10^\circ \cdot \tan 20^\circ = \tan 30^\circ = \tan 15^\circ = 2-\sqrt{3}$

(97) $\sin\theta\cdot\cos\theta$ का max. value = ? ($\theta + \alpha + \theta = 45^\circ$) $(1/2)$

→ $\frac{2\sin\theta\cdot\cos\theta}{2} = \frac{\sin 2\theta}{2}$ ⇒ $\sin 2\theta$ max. value = 1

so $\frac{1}{2}$ = max value of $\sin\theta\cdot\cos\theta$.

(98) $\tan 15^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 75^\circ = ?$ (1)

→ $\tan 15^\circ \times \tan 25^\circ \times 1 \times \tan(90-25)\times\tan(90-15)$

→ $\tan 15^\circ \times \tan 25^\circ \times 1 \times \cot 25^\circ \times \cot 15^\circ = 1$

$\rightarrow q(p^2-1) \Rightarrow (\sec\theta+\operatorname{cosec}\theta)[(\sin\theta+\cos\theta)^2-1]$

$= \left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)\left[\underbrace{\sin^2\theta+\cos^2\theta}_{1}+2\sin\theta\cos\theta-1\right]$

$\Rightarrow \frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}(2\sin\theta\cos\theta) = 2(\sin\theta+\cos\theta) = \underline{2p}$

OR

T10 ~~Take~~

Take any value of θ like 30, 45, 60° etc. & then find value of p, q etc.

Let $\theta = 45° \Rightarrow p = \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$, $q = \sqrt{2}+\sqrt{2} = \underline{2\sqrt{2}}$

$q(p^2-1) \Rightarrow 2\sqrt{2}((\sqrt{2})^2-1) = 2\sqrt{2}(1) = \underline{2\sqrt{2}} = \underline{\underline{2p}}$

---

(31) if $\operatorname{cosec}\theta-\sin\theta = l$ and $\sec\theta-\cos\theta = m$ then $\underline{l^2m^2(l^2+m^2+3)}$ = ?

(1)

$\rightarrow$ Let $\theta = 45°$, $l = \sqrt{2}-\frac{1}{\sqrt{2}} = \frac{2-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$, $m = \frac{1}{\sqrt{2}}$

$l^2m^2(l^2+m^2+3) \rightarrow \frac{1}{2}\times\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}+3\right) = \frac{1}{4}\times\frac{8}{2} = 1$

---

(32) $x\sin^3\theta+y\cos^3\theta = \sin\theta\cdot\cos\theta$ and $x\sin\theta = y\cos\theta$ then $\underline{x^2+y^2}$ = ?

(1)

$\rightarrow$ Let $\theta = 45° \Rightarrow x\cdot\frac{1}{2\sqrt{2}}+y\cdot\frac{1}{2\sqrt{2}} = \frac{1}{2}$ and $\frac{x}{\sqrt{2}}-\frac{y}{\sqrt{2}} = 0$

$\frac{x+y}{2\sqrt{2}} = \frac{1}{2} \Rightarrow \underline{x+y=2\sqrt{2}}$ and $\frac{x-y}{\sqrt{2}} = 0 \Rightarrow x-y=0$

$x+y=2\sqrt{2}$, $x-y=0$

$2x=2\sqrt{2} \rightarrow x = \frac{2\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$; $2y=2\sqrt{2} \rightarrow y = \frac{1}{\sqrt{2}}$

$x^2+y^2 = \left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2 \rightarrow \frac{1}{2}+\frac{1}{2} = 1$

① if $\frac{3\sin\theta - 2\cos\theta}{3}$ = 2

$\left[\frac{\sin\theta}{\cos\theta} = \frac{4}{3} \Rightarrow 3\sin\theta = 4\cos\theta \text{ so } \frac{6\cos\theta}{2\cos\theta} = 3\right]$

② if $\tan\theta + \cot\theta = 2$ then $\tan^5\theta + \cot^{10}\theta = ?$ (2)

$\rightarrow \left[\theta = 45° \Rightarrow (1)^5 + (1)^{10} = 2\right]$

③ if $\sin\theta - \cos\theta = \frac{7}{13}$ then $\sin\theta + \cos\theta = ?$ (17/13)

$\left[\boxed{17}\ \sin\theta + \cos\theta = \sqrt{2 - x^2} \Rightarrow \sqrt{2 - \frac{49}{169}} = \frac{17}{13}\right]$

④ if $\sin\alpha + \cos\beta = 2$ then $\sin\left(\frac{2\alpha+\beta}{3}\right) = ?$ ($\sqrt{3}/2$)

$\left[\alpha = 90° \quad \beta = 0° \quad \sin\left(\frac{180}{3}\right) = \sin 60° = \frac{\sqrt{3}}{2}\right]$

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⑤ if $\cos^4\theta - \sin^4\theta = \frac{2}{3}$ then $2\cos^2\theta - 1 = ?$ (2/3)

$\left[(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}\right.$

$\left.\cos^2\theta - \sin^2\theta = \frac{2}{3} \Rightarrow 2\cos^2\theta - 1 = \frac{2}{3}\right]$

⑥ $\cot 10° \cdot \cot 20° \cdot \cot 60° \cdot \cot 70° \cdot \cot 80° = ?$ ($1/\sqrt{3}$)

$\left[\cot 10° \cdot \tan 10° \cdot \cot 20° \cdot \tan 20° \cdot \cot 60° \rightarrow \frac{1}{\sqrt{3}}\right]$

⑦ if $\sin\alpha \sec(30+\alpha) = 1$ then $\sin\alpha + \cos 2\alpha = ?$ (1)

$\left[\sin\alpha = \cos(30+\alpha) \Rightarrow \sin\alpha = \sin[90 - (30+\alpha)]\right.$

$\left.\alpha = 90 - 30 - \alpha \Rightarrow \alpha = 30° \Rightarrow \sin 30° + \cos 60° = \frac{1}{2} + \frac{1}{2} = 1\right]$

⑧

$\tan^2\theta = \frac{1}{3} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30$

(hint) at $\theta = 30$

(9) $\sin^2 1' + \sin^2 5' + \sin^2 9' + \cdots + \sin^2 85' + \sin^2 89' = ?$

$(\sin^2 1' + \sin^2 89') + (\sin^2 5' + \sin^2 85') + \cdots \sin^2 45'$

1, 5, 9, ---- 89 $\rightarrow$ $89 = 1 + (n-1)4 \Rightarrow 89 = 1 + 4n - 4 \Rightarrow n = 23$

So 11 pairs & $\sin^2 45'$ $\quad$ OR $\quad 89/4 = 22 + 1$ or 11 pairs & $\sin 45'$

$\rightarrow (1+1+1+1+1+1+1+1+1+1+1) + \left(\frac{1}{\sqrt{2}}\right)^2 \rightarrow 11 + \frac{1}{2} = 11\frac{1}{2}$

(OR) $\frac{89-1}{4} = \frac{88}{4} = 22 = 11$ pairs $+ \sin^2 45'$

(10) if $\sin\theta + \operatorname{cosec}\theta = 2$ then $\sin^5\theta + \operatorname{cosec}^5\theta = ?$ (2)

$\theta = 90°$ so $\sin^5\theta + \operatorname{cosec}^5\theta = 1^5 + 1^5 = 2$

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(11) $\sin^2 5' + \sin^2 10' + \cdots + \sin^2 85' + \sin^2 90' = ?$ $\quad (9\frac{1}{2})$

5, 10, --- 85 $+ (90) \Rightarrow \frac{85}{5} \rightarrow 17 \rightarrow$ 8 pairs $+ 45' + 90'$

$\Rightarrow (\sin^2 5' + \cos^2 5') \cdots \Rightarrow 8 \times 1 + \frac{1}{2} + 1 \Rightarrow 9\frac{1}{2}$

$\frac{85-5}{5} = 8$ pairs $+ \sin^2 45' + \sin^2 90'$

(12) if $\tan 2\theta \cdot \tan 4\theta = 1$ then $\tan 3\theta = ?$ (a)

$\tan 2\theta = \frac{1}{\tan 4\theta} = \cot 4\theta \Rightarrow \tan 2\theta = \tan(90 - 4\theta) \Rightarrow 6\theta = 90 \Rightarrow \theta = 15'$

$\tan 3\theta = \tan 45 = 1$

(13) if $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$ then $2\sin\theta + \cos\theta = ?$ $\quad (3/\sqrt{2})$

$\theta = 45°$ then $2 \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ so $2 \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

(hint)

(14) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$ then $\sin^4\theta - \cos^4\theta = ?$ $\quad (3/5)$

$2\sin\theta = 4\cos\theta \Rightarrow \sin\theta = 2\cos\theta \Rightarrow \tan\theta = 2$

2, $\sqrt{5}$, $\sin\theta = \frac{2}{\sqrt{5}}$

(15) If $1+\tan^2\theta+\tan^4\theta=1$ (0)

[$1+\tan^2\theta+\tan^4\theta=1 \Rightarrow 2\tan^2\theta=0 \Rightarrow \tan^2\theta=0 \Rightarrow \theta=0$]

---

(16) if $\cos^2\alpha+\cos^2\beta=2$ then $\tan^3\alpha+\sin^5\beta=?$ (0)

[$\alpha=0, \beta=0 \rightarrow \tan^3 0+\sin^5 0=0$]

(n&t)

---

(17) $\sin\theta+\cos\theta=\frac{17}{13}$ then $\sin\theta-\cos\theta=?$ $(\frac{7}{13})$

[$\sqrt{2-x^2}=\sqrt{2-\frac{289}{169}}=\sqrt{\frac{338-289}{169}}=\sqrt{\frac{49}{169}}=\frac{7}{13}$]

---

(18) if $\tan\theta\cdot\tan 2\theta=1$ then $\sin^2 2\theta+\tan^2\theta=?$ $(15/4)$

[$\tan\theta=\cot 2\theta \equiv \tan(90-2\theta) \Rightarrow 3\theta=90 \Rightarrow \theta=30$

$\sin^2 60+\tan^2 60 \Rightarrow \frac{3}{4}+\frac{3}{1} \rightarrow \frac{15}{4}$]

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(19) $2\sin^2\theta+3\cos^2\theta$ का minimum value = ? (2)

[if $\theta=0 \rightarrow 2\times 0+3\times 1=3$

if $\theta=90 \rightarrow 2\times 1+3\times 0=2$]

---

(20) If $\operatorname{cosec} 39=x$ then $\frac{1}{\operatorname{cosec}^2 51}+\sin^2 39+\tan^2 51-\frac{1}{\sin^2 51\sec^2 39}=?$ (x^2-1)

[$\frac{\sin^2 51+\sin^2 39}{1}+\cot^2 39-\frac{1}{\cos^2 39\cdot\sec^2 39} \Rightarrow 1+\cot^2 39-1 \Rightarrow \operatorname{cosec}^2 39-1$

$=x^2-1$]

---

(21) if $\frac{\tan\theta+\cot\theta}{\tan\theta-\cot\theta}=2$ then $\sin\theta=?$ $(\sqrt{3}/2)$

[$\tan\theta=3\cot\theta \Rightarrow \tan^2\theta=3 \Rightarrow \tan\theta=\sqrt{3} \Rightarrow \theta=60$

$\sin 60=\frac{\sqrt{3}}{2}$]

(22) $[\tan(2\theta+45) = \tan(90-3\theta) \Rightarrow 5\theta = 45 \Rightarrow \theta = 9]$

(23) if $\sec^2\theta + \tan^2\theta = \frac{7}{12}$ then $\sec^4\theta - \tan^4\theta = ?$ (7/12)

$[(\sec^2\theta + \tan^2\theta)(\underbrace{\sec^2\theta - \tan^2\theta}_{1}) \Rightarrow \frac{7}{12} \times 1 = \frac{7}{12}]$

(24) if $\tan A = -\frac{1}{3}$ and $\tan B = -\frac{1}{2}$ then $A+B = ?$ (135°)

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} \Rightarrow \frac{-\frac{1}{3} + \frac{1}{2}}{1-(-\frac{1}{3})(-\frac{1}{2})} \Rightarrow \frac{\frac{-2-3}{6}}{1-\frac{1}{6}} \Rightarrow -\frac{5}{6} \times \frac{6}{5} = -1$

$\tan(A+B) = \underbrace{\tan(135)}_{-1} \Rightarrow A+B = 135°$

(25) $\sin(A+B) = \frac{1}{2}$ and $\cos(A-B) = 1$ then $\frac{A}{B} = ?$ (3/1)

$A+B = 30$
$A-B = 0 \Rightarrow A = \frac{30}{2} = 15°, \quad B = 15°$

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(26) if $A+B = 45°$ then $(1+\tan A)(1+\tan B) = ?$ (2)

$\tan(A+B) = \tan 45° \Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1 \Rightarrow \tan A + \tan B = 1 - \tan A \cdot \tan B$

Adding (1) both sides $\Rightarrow 1 + \tan A + \tan B + \tan A \tan B = 2$

$(1+\tan A) + \tan B(1+\tan A) = 2$

$(1+\tan A)(1+\tan B) = 2$

(27) $\tan 720° - \cos 270° - \sin 150° \cdot \cos 120° = ?$ (1/4)

$\tan(2\times 360 + 0) - \cos(360-90) - \sin(90+60) \cdot \cos(90+30)$

$\tan 0° - \cos 90 + \cos 60° \cdot \sin 30°$

$0 - 0 + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

① [illegible]

$A-B=30^\circ$, $A+B=60^\circ$ $\Rightarrow$ $A=45^\circ$, $B=15^\circ$

---

② $\cos 20^\circ + \cos 40^\circ + \cos 60^\circ + \cdots + \cos 160^\circ + \cos 180^\circ = ?$

$\cos 180^\circ = -1$, $\cos 160^\circ = \cos(180-20) = -\cos 20^\circ$, $\cos 140^\circ = -\cos 40^\circ$ etc.

So $\cos 20^\circ + \cos 40^\circ + \cos 60^\circ + \cdots - \cos 60 - \cos 40 - \cos 20 - 1$

$= \boxed{-1}$

③ if $\cot A + \operatorname{cosec} A = 3$ & $0 \le A \le 90^\circ$ then $\cos A$?

$\operatorname{cosec}^2 A - \cot^2 A = 1$

$(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A) = 1$

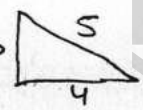
$\operatorname{cosec} A - \cot A = \frac{1}{3}$

$\operatorname{cosec} A + \cot A = 3$

$\operatorname{cosec} A - \cot A = \frac{1}{3}$

$2\operatorname{cosec} A = \frac{10}{3}$

$\operatorname{cosec} A = \frac{5}{3}$



So $\boxed{\cos A = \frac{4}{5}}$

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④ if $\tan 2\theta \cdot \tan 4\theta = 1$ then $\tan 3\theta = ?$

$\tan 2\theta = \frac{1}{\tan 4\theta} = \cot 4\theta$

$\tan 2\theta = \tan(90-4\theta)$

$6\theta = 90 \Rightarrow \theta = 15^\circ \Rightarrow \boxed{\tan(3\times 15) = 1}$

Or when $\tan\theta_1 \cdot \tan\theta_2 = 1$ then $\theta_1 + \theta_2 = 90^\circ$ always

⑤ $\sin^2 1 + \sin^2 3 + \sin^2 5 + \cdots \sin^2 87 + \sin^2 89 = ?$

$\sin^2 89 = \cos^2 1$

$\sin^2 87 = \cos^2 3$

So $(\sin^2 1 + \sin^2 89) + (\sin^2 3 + \sin^2 87) + \cdots + \sin^2 45$

$\downarrow$ 1 $\qquad$ $\downarrow$ 1

Short cut $\rightarrow \frac{\text{Last} - \text{Ist}}{\text{common diff}} \Rightarrow \frac{89-1}{2} \Rightarrow 44 + 1 \rightarrow \sin^2 45^\circ$

So 22 Pairs + $\sin^2 45^\circ$

$22 + \frac{1}{2} = \boxed{22\frac{1}{2}}$

⑥ → [$1+1+1=3$; $\sin\theta=1 \Rightarrow \theta=90°$; $\cos90+\cos90+\cos90° = \boxed{0}$]

⑦ if $\sin\theta+\sin^2\theta=1$ then $\cos^2\theta+\cos^4\theta=?$

[$\sin\theta=1-\sin^2\theta=\cos^2\theta \rightarrow \sin^2\theta+\sin^2\theta=\boxed{1}$]

Always

⑧ if $\sin n+\sin^2 n=1$ then $\cos^8 n+2\cos^6 n+\cos^4 n=?$

↓

①

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⑨ if $\sin n+\sin^2 n=1$ then $\cos^{12} n+3\cos^{10} n+3\cos^8 n+\cos^6 n-1$?

↓

① $-1=\boxed{0}$ Ans.

⑩ if $\cos A+\cos^2 A=1$ then $\sin^2 A+\sin^4 A=?$

↓

①

---

⑪ Find max. value of $\sin^4\theta+\cos^4\theta=?$

[max. value of $\sin^{2n}\theta+\cos^{2m}\theta=1$]

⑫ Find min. value of $\sin^2\theta+\operatorname{cosec}^2\theta+\cos^2\theta+\sec^2\theta$?

[$\operatorname{cosec}^2\theta+\sec^2\theta+1$

min value of $a\operatorname{cosec}^2\theta+b\sec^2\theta=(\sqrt{a}+\sqrt{b})^2$ so $(\sqrt{}+\sqrt{})^2+1=\boxed{5}$]

⑬ Find max. value of $\sin^3 n.\cos^3 n$?

[$\left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^3=\boxed{\frac{1}{8}}$]

---

⑭ Min. value of $\tan^2\theta+\cot^2\theta=?$

[$2\sqrt{ab} \rightarrow 2\sqrt{1\times1}=\boxed{2}$]

(15)

At $\theta = 0^\circ \rightarrow 0 + 1 = \boxed{1} \rightarrow$ max. value

At $\theta = 90^\circ \rightarrow 1 + 0 = 1$

At $\theta = 30^\circ \rightarrow \left(\frac{1}{2}\right)^6 + \left(\frac{\sqrt{3}}{2}\right)^6 \rightarrow \frac{1}{64} + \frac{27}{64} \Rightarrow \frac{28}{64} = \frac{7}{16}$

At $\theta = 60^\circ \rightarrow \left(\frac{\sqrt{3}}{2}\right)^6 + \left(\frac{1}{2}\right)^6 \rightarrow \frac{7}{16}$

At $\theta = 45^\circ \rightarrow \left(\frac{1}{\sqrt{2}}\right)^6 + \left(\frac{1}{\sqrt{2}}\right)^6 \rightarrow \frac{1}{8} + \frac{1}{8} \Rightarrow \boxed{\frac{1}{4}} \rightarrow$ min. value

(16) Max. value of $81^{\sin n} \cdot 27^{\cos n} = ?$

$3^{4\sin n} \cdot 3^{3\cos n} \rightarrow 3^{4\sin n + 3\cos n} \longrightarrow 3^5 = \boxed{243}$

Max. value of $4\sin n + 3\cos n \Rightarrow \sqrt{4^2 + 3^2} = 5$

(17) if $\tan\theta + \cot\theta = 2$ then $\tan^{17}\theta + \cot^{19}\theta = ?$ $\rightarrow \boxed{2}$ Ans.

(18) if $\frac{\cos A}{\cos B} = m$ and $\frac{\cos A}{\sin B} = n$ then $(m^2 + n^2)\cos^2 B = ?$

Put $A = B = 60^\circ \longrightarrow m = 1,\ n = \frac{1}{\sqrt{3}} \Rightarrow \left(1 + \frac{1}{3}\right) \times \cos^2 60^\circ$

At 90° or $0^\circ \Rightarrow \sin 0^\circ = 0$, $\cos 90^\circ = 0$ & $\frac{n}{0} =$ N.D.

$\frac{4}{3} \times \frac{1}{4} = \boxed{\frac{1}{3}}$

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(19) if $\cos\theta + \sec\theta = \sqrt{3}$ then $\cos^3\theta + \sec^3\theta = ?$

$n + \frac{1}{n} = \sqrt{3}$ | $n^3 + \frac{1}{n^3} = 1 < 3 - 3 1 <$ ⟩ Algebra shortcut

$\cos\theta + \frac{1}{\cos\theta} = \sqrt{3}$

$\cos^3\theta + \frac{1}{\cos^3\theta} = (\sqrt{3})^3 - 3\sqrt{3} = \boxed{0}$

(20) if $\sin(n+y) = \cos[3(n+y)]$ then $\tan[2(n+y)] = ?$

$\sin(n+y) = \sin[90 - 3(n+y)]$

$n + y = 90 - 3n - 3y$

$4n + 4y = 90$

$n + y = \frac{90}{4} = 45$

$\tan[2(n+y)] \rightarrow \tan 45^\circ = \boxed{1}$

IM

① If $A+B+C=180$

then $\boxed{\tan A+\tan B+\tan C=\tan A\cdot\tan B\cdot\tan C}$

② if $A+B+C=90^\circ$

then $\boxed{\cot A+\cot B+\cot C=\cot A\cdot\cot B\cdot\cot C}$

③
$$\sin\theta\cdot\sin(60-\theta)\sin(60+\theta)=\frac{3}{4}\sin 3\theta$$

or

$$\sin\theta_1\cdot\sin\theta_2\cdot\sin\theta_3=\frac{3}{4}\sin 3\theta_1$$

[for $\theta_1+\theta_2=60^\circ$, $\theta_2+\theta_3=120^\circ$]

$$\cos\theta_1\cdot\cos\theta_2\cdot\cos\theta_3=\frac{3}{4}\cos 3\theta_1$$

$$\tan\theta_1\cdot\tan\theta_2\cdot\tan\theta_3=\tan 3\theta_1$$

④ (i) if $\sin\theta+\operatorname{cosec}\theta=2$ then $\sin^n\theta+\operatorname{cosec}^m\theta=2$
where n, m are any numbers.

(ii) if $\cos\theta+\sec\theta=2$ then $\cos^n\theta+\sec^m\theta=2$

(iii) if $\tan\theta+\cot\theta=2$ then $\tan^n\theta+\cot^m\theta=2$

⑤
$$\sin 15^\circ=\cos 75^\circ=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

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$$\sin 75^\circ=\cos 15^\circ=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\tan 15^\circ=\frac{\sin 15^\circ}{\cos 15^\circ}=2-\sqrt{3}$$

$$\tan 75^\circ=\frac{\sin 75}{\cos 75}=2+\sqrt{3}$$

$\tan 22\frac{1}{2}^\circ=\sqrt{2}-1$

$\cot 22\frac{1}{2}^\circ=\sqrt{2}+1$

$\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5}+1}{4}$

$\sin 22\frac{1}{2}^\circ = \frac{\sqrt{2-\sqrt{2}}}{2}$, $\cos 22\frac{1}{2}^\circ = \frac{\sqrt{2+\sqrt{2}}}{2}$

---

* MAX. & MIN. values:-

① Max. value of $a\sin\theta \pm b\cos\theta = \sqrt{a^2+b^2}$

Min. value $= -\sqrt{a^2+b^2}$

② $\sin^{2n}\theta + \cos^{2m}\theta \leq 1$

or max. value of $\sin^{2n}\theta + \cos^{2m}\theta$ is 1

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③ Max. value of $\sin^n\theta \cdot \cos^n\theta = \left(\frac{1}{2}\right)^n$

④ Use $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ in case of $\sin^2\theta \leftrightarrow \cos^2\theta$ identities. to find max. & min. values.

⑤ Min. value of $a\tan^2\theta + b\cot^2\theta = 2\sqrt{ab}$

⑥ Min. value of $a\sec^2\theta + b\operatorname{cosec}^2\theta = (\sqrt{a}+\sqrt{b})^2$

⑦ Min. value of $a\sec^2\theta + b\cos^2\theta = 2\sqrt{ab}$

⑧ Min. value of $a\operatorname{cosec}^2\theta + b\sin^2\theta = 2\sqrt{ab}$

---

* if $0 < \theta < 90^\circ$ & $\theta_1 > \theta_2$ then

$\sin\theta_1 > \sin\theta_2$

$\tan\theta_1 > \tan\theta_2$

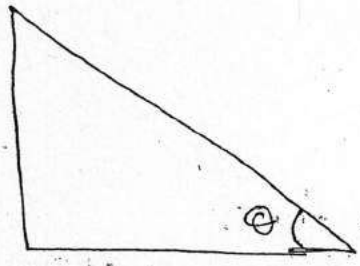
$\sec\theta_1 > \sec\theta_2$

if $0 < \theta < 90^\circ$ & $\theta_1 < \theta_2$ then

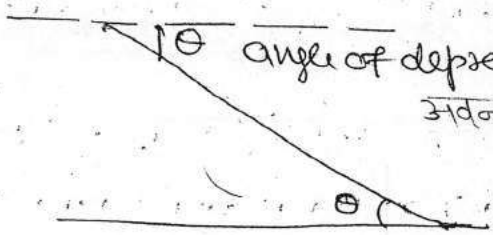
$\cos\theta_1 > \cos\theta_2$

$\cot\theta_1 > \cot\theta_2$

$\operatorname{cosec}\theta_1 > \operatorname{cosec}\theta_2$

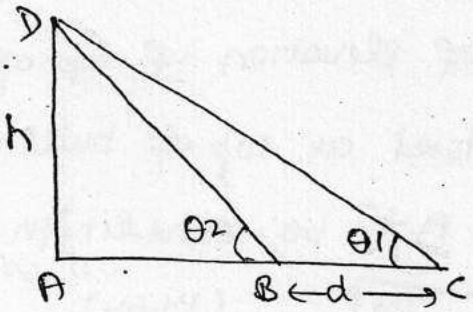


Angle of elevation
उन्नयन कोण



θ angle of depression
अवनमन कोण

	0°	30°	45°	60°	90°
sin	0	1/2	1/√2	√3/2	1
cos	1	√3/2	1/√2	1/2	0
tan	0	1/√3	1	√3	∞
Cot	∞	√3	1	1/√3	0



T1

$$d = h(\cot\theta_1 - \cot\theta_2)$$

$\theta_2 > \theta_1$

h = height.

$d = AC - AB$

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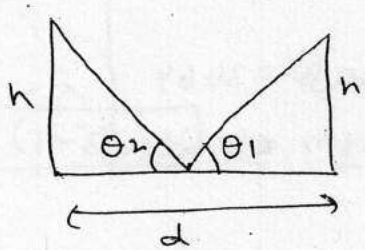
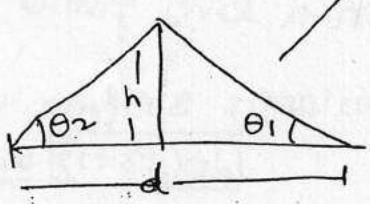


T2

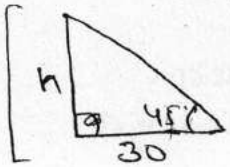
$$d = h(\cot\theta_1 + \cot\theta_2)$$

or



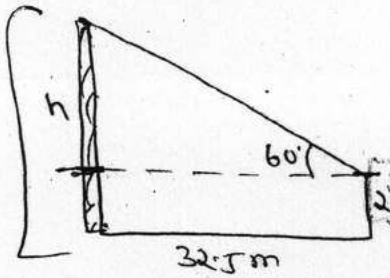
Q①

pt. which is 30 m away from tower? (30m)



$\tan 45° = \frac{h}{30} \rightarrow h = 30$

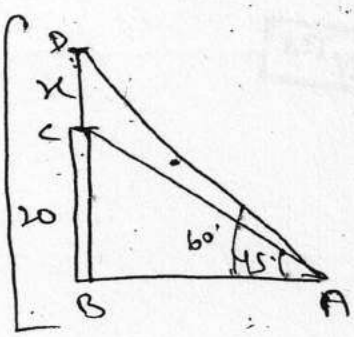
② A man 2.5 m tall is 32.5 m away from a building. The angle of elevation of top of the building from his eyes is 60°. Find the ht of building? (58.79 m)



$\tan 60° = \frac{h}{32.5} \rightarrow h = 32.5 \times \sqrt{3} = 56.29$ m

Total ht. $= 56.29 + 32.5 = 58.79$ m

③ From a pt. A on the ground, the angle of elevation of top of a 20 m tall building is 45°. A flag is hoisted at top of building and angle of elevation of top of flag from A is 60°. Find length of flag staff? (14.64)

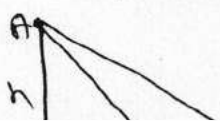


$\tan 45° = \frac{20}{AB} \rightarrow AB = 20$ m

$\tan 60° = \frac{20+x}{20} \rightarrow x + 20 = 20\sqrt{3} = 34.64$

$x = 14.64$ m or $20(\sqrt{3}-1)$

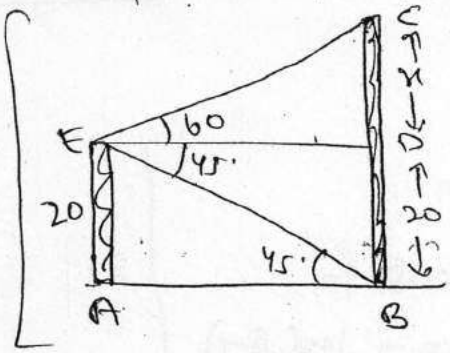
④ The shadow of a building standing on a level ground is found to be 40 m longer when sun's altitude becomes 30° from 45°. Find ht. of building? $20(\sqrt{3}+1)$ or (54.64 m)



$\tan 45° = \frac{AB}{BC} \rightarrow BC = AB$

$\tan 30° = \frac{AB}{AB} \rightarrow \frac{AB}{AB} = \frac{1}{\sqrt{3}} \rightarrow AB = 40 = 54.675$ m

(5) ... of the top of a tower is 60° and the angle of depression of its foot is 45°. Find ht. of tower?



$BC = x + 20$

$AB = \tan 45° \times 20 \Rightarrow 20 = ED$

$\tan 60° = \frac{x}{20} \Rightarrow x = 20\sqrt{3} = 34.64$

Ht. $= 34.64 + 20 = 54.64$ m

$= 20(\sqrt{3}+1)$

(54.64 m) or $20(\sqrt{3}+1)$

(6) A tree is broken and its top makes an angle of 60° with ground. and distance between the fallen part and root of tree is 12 m. Find the length of tree before break? total

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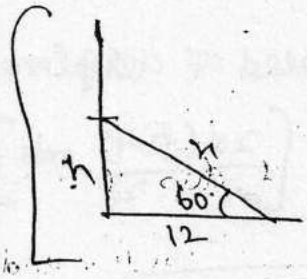
$12(\sqrt{3}+2)$



$\frac{h}{12} = \tan 60° \rightarrow h = 12\sqrt{3}$

$x = \sqrt{(12\sqrt{3})^2 + (12)^2} = \sqrt{576} = 24$ or $\cos 60° = \frac{12}{x} \rightarrow x = 24$ m

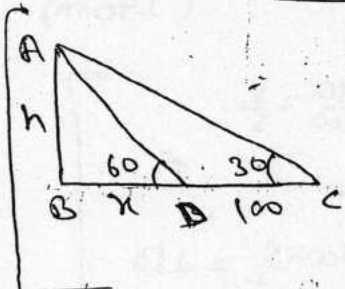
Length of tree $= 12\sqrt{3} + 24 = 12(\sqrt{3}+2)$ m.

(7) The angle of elevation of the top of a tower from a pt. on the ground is 30°. It inc. to 60° when pt. moves 100 m towards tower. Find ht. of the tower?

(50√3)

$d = h(\cot\theta_1 - \cot\theta_2) \rightarrow h = \frac{100}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \frac{100\sqrt{3}}{2} = 50\sqrt{3}$

or



$\tan 60° = \frac{h}{x} \rightarrow h = \sqrt{3}x \rightarrow x = \frac{h}{\sqrt{3}}$

$\tan 30° = \frac{h}{x+100} \rightarrow x + 100 = \sqrt{3}h \rightarrow \sqrt{3}h - \frac{h}{\sqrt{3}} = 100$

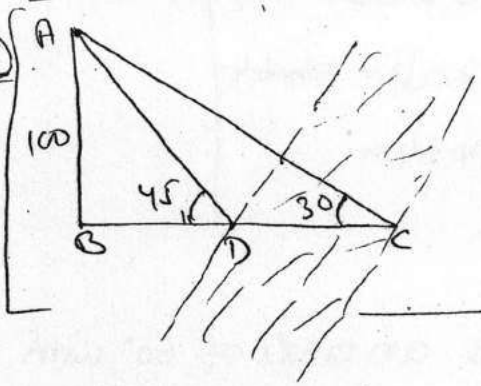
$\frac{3h-h}{\sqrt{3}} = 100 \rightarrow 2h = 100\sqrt{3}, h = 50\sqrt{3}$

(8) ... the both ... 30 and 45. If ht. of tower is 100 m then find the breadth of river?

$100(\sqrt{3}-1)$

$d = h((\cot\theta_1 - \cot\theta_2) \rightarrow 100(\sqrt{3}-1)$

(OR)

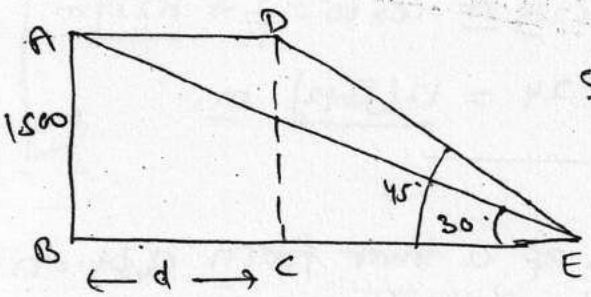


$\tan 45 = \frac{AB}{BD} \rightarrow BD = 100$ m

$\tan 30 = \frac{AB}{BC} \rightarrow BC = 100\sqrt{3}$ m

$CD = BC - BD = 100\sqrt{3} - 100 \rightarrow 100(\sqrt{3}-1)$

(9) An ~~aer~~ aeroplane is flying ⊥ to the ground at a ht. of 1500m. An observer gets angle of elevation as 30° and after 2 mins it is 45°. if aeroplane is coming towards observer then find speed of aeroplane?



$d = h(\cot\theta_1 - \cot\theta_2)$

$= 1500(\sqrt{3}-1)$

$\text{speed} = \frac{1500(\sqrt{3}-1)}{120 \text{ (secs)}} = \frac{25(\sqrt{3}-1)}{2}$ m/s.

$\left[\frac{25(\sqrt{3}-1)}{2} \text{ m/s}\right]$

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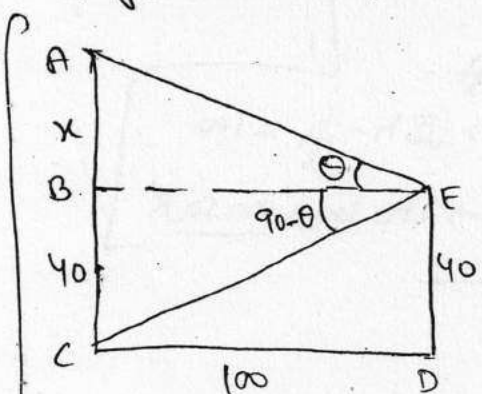
(10) The ht. of a building which is 100 m away from a tower is 40 m. A right angle is made at the top of building by the top & bottom of tower. Find ht of tower?

(290 m)



$\Delta BCE \rightarrow \tan(90-\theta) = \frac{40}{100} = \frac{2}{5}$

$\cot\theta = \frac{2}{5}$, $\tan\theta = \frac{5}{2}$

$\Delta ABE \rightarrow \tan\theta = \frac{x}{100} \Rightarrow x = 100 \times \frac{5}{2} = 250$

ht of tower = $250 + 40 = 290$ m

System of equations:-

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

1) Unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

2) Infinite solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

] Consistent System

3) No solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$] Inconsistent system.

*HOMOGENEOUS system

$a_1x + b_1y = 0$

$a_2x + b_2y = 0$

1) only solution i.e. ~~a=0~~ $x=0, y=0$, when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

2) Infinite solution when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

* Graphs of $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ will be:-

1) Parallel if system has no solution. ⟶ ⟶

2) Coincident if infinite solutions. ⟶⟶

3) Intersecting if unique solution ✕

$b=2$

Q From ① $2^a \cdot 2^a - 3 \cdot 3^b = 5 \rightarrow 4 \cdot 2^a - 3 \cdot 3^b = 5$ let $2^a = x$, $3^b = y$

so $x+y=17$ — ① solving $x = 8 = 2^a \rightarrow \boxed{a=3}$

$4x-3y=5$ — ② $y = 9 = 3^b \rightarrow \boxed{b=2}$

---

② For what value of k will the equations $x+2y+6=0$ and $3x+ky+18=0$ represent coincident lines? (infinite sols) (6)

$\frac{1}{3} = \frac{2}{k} = \frac{6}{18} \rightarrow k=6$

---

③ For what value of k the system of eqns $x+3y+7=0$ and $4x+ky+19=0$ has no solution? (12)

$\frac{1}{4} = \frac{3}{k} \neq \frac{7}{19} \rightarrow k=12$

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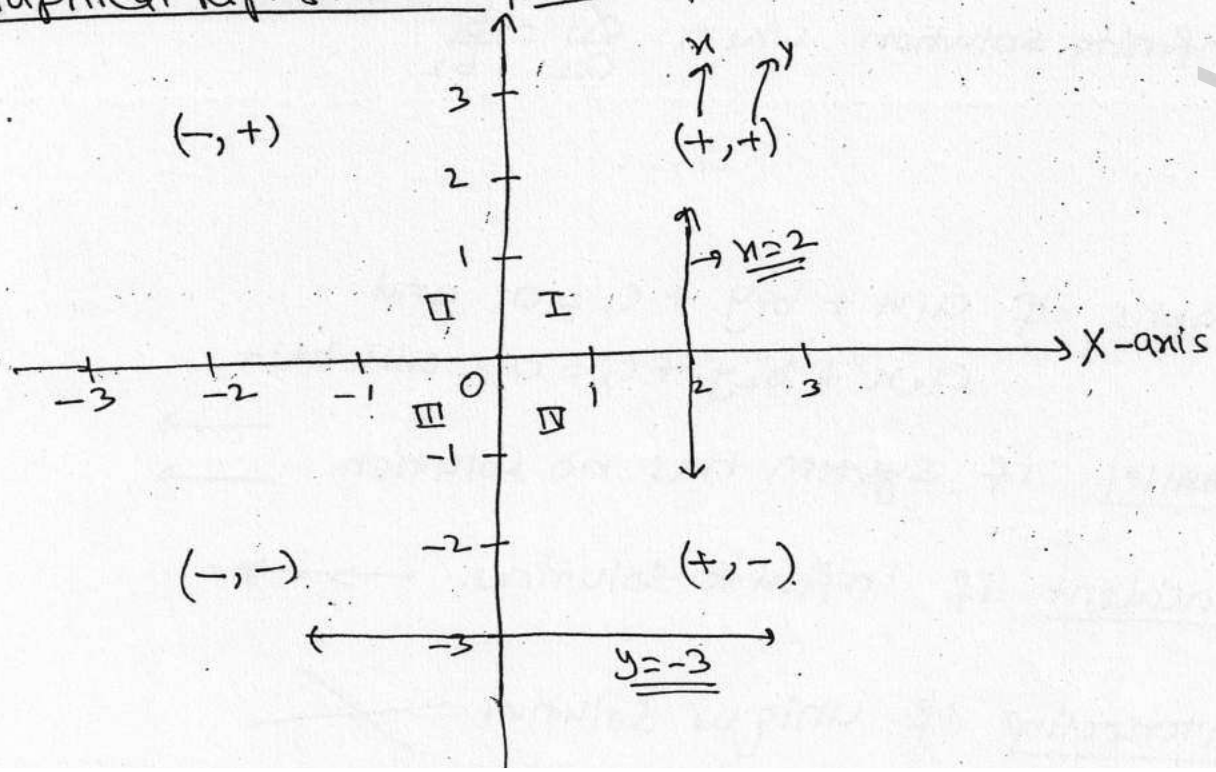
④ Find value of k so that the lines $x+3y-8=0$ & $kx+12y+5=0$ are parallel? (4)

means No solution

$\frac{1}{k} = \frac{3}{12} \neq \frac{-8}{5} \rightarrow k=4.$

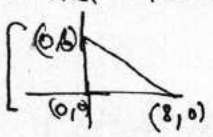
---

* Graphical Representation



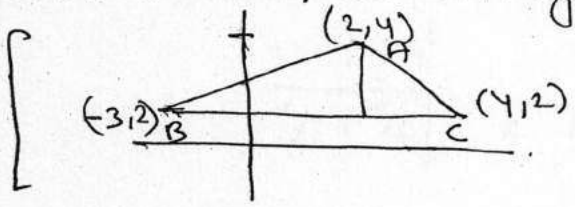
① [0, -2]

② Find the Area of a △OAB with O(0,0), A(8,0), B(0,6)? [24]



$\frac{1}{2} \times 8 \times 6 = 24$ sq. units

③ Find Area of △ having vertices A(2,4), B(-3,2) & C(4,2)?



AD = 4 − 2 = 2

BC = 4 + 3 = 7

$A = \frac{1}{2} \times 7 \times 2 = 7 \text{ unit}^2$

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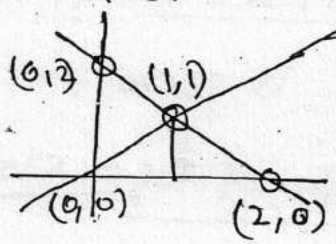
④ Find the area of △ formed by the graph of lines $x - y = 0$, $x + y = 2$ and x-axis? (1 unit²)

$x - y = 0$ | $x = y$ | $x \geq 0, y \geq 0$ | $x \geq 2, y \geq 2$

$x + y \geq 2$ | $x \geq 0, y \geq 2$ | $y \geq 0, x \geq 2$

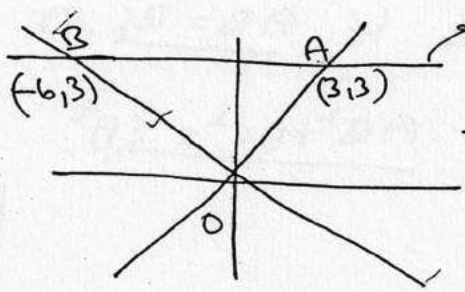


$\frac{1}{2} \times 2 \times 1 = 1 \text{ unit}^2$

⑤ Find the area of region bounded by lines $x - y = 0$, $x + 2y = 0$ and $y = 3$? (13.5 unit²)

$x - y = 0$ or $x = y$ | $x \geq 0, y \geq 0$ | $x \geq 3, y = 3$

$x + 2y = 0$ | $x \geq 0, y \geq 0$ | $y = 3, x = -6$



$\frac{1}{2} \times 9 \times 3 = 13.5$

⑥ Find the area of △ formed by graph of $ax + by = c$ and coordinate axes? ($\frac{c^2}{2ab}$ unit²)

$x = 0$ ⇒ Putting in eqn | $y = 0$

$y = 0$ | $0 + by = c$

$y = \frac{c}{b}$

$y = 0$

$ax + 0 = c$, $x = \frac{c}{a}$

Area $= \frac{1}{2} \times x \times y$

$= \frac{c^2}{2ab}$

X-Coordinate → Abscissa ← (3,5)

Y-Coordinate → ordinate ←

Distance formula → A(x_1,y_1) ——— B(x_2,y_2)

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

* Distance of P(x,y) from origin. $= \sqrt{x^2+y^2}$

✠ FOR GIVEN THREE POINTS — A, B and C :-

① Collinearity :- → if AB+BC = AC or AB+AC = BC or AC+BC = AB

→ if Area of Δ ABC is zero.

→ slope of AB = slope of BC = slope of CA

② Vertices of a Δ

→ equilateral Δ if AB = BC = CA

→ Isosceles Δ if AB = BC or BC = CA or CA = AB

→ rt Δ if $AB^2+BC^2 = CA^2$

③

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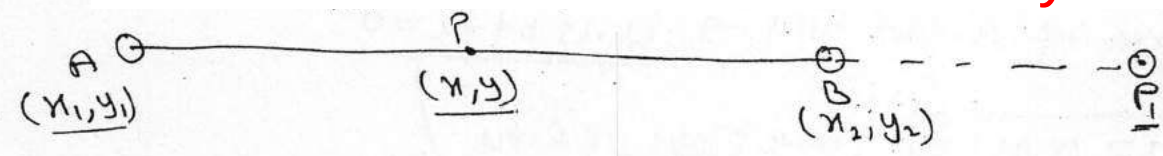
Note → four given Pts are collinear if area of □ is zero.

* Sect



Internally → $x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

$\left[\frac{AP}{PB} = \frac{m}{n}\right]$

[~~for internally~~ if m:n is +ve then internally]

Externally → $x = \frac{mx_2 - nx_1}{m-n}$, $y = \frac{my_2 - ny_1}{m-n}$

$\left[\frac{AP_1}{BP_1} = -\frac{m}{n}\right]$

[For externally m:n will come out to be −ve]

→ Co-ordinates of mid pt → $m:n \to 1:1$

So $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

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* TRIANGLE:-

① Area of Δ:- Vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$

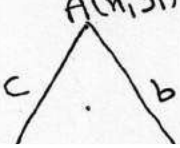
$$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\text{Area of } \Delta = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

② Co-ordinates of Centroid

$$x = \frac{x_1 + x_2 + x_3}{3}, \quad y = \frac{y_1 + y_2 + y_3}{3}$$

③ Co-ordinates of Incentre



$$x = \frac{ax_1 + bx_2 + cx_3}{a+b+c}, \quad y = \frac{ay_1 + by_2 + cy_3}{a+b+c}$$

* St

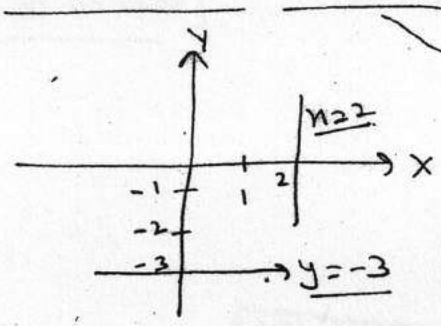
① General form of a st line → $ax+by+c=0$

or $\boxed{y=mx+c}$ m → slope of line

So $by = -ax-c$

$y = \left(\frac{-a}{b}\right)x - \frac{c}{b}$ $\Rightarrow \boxed{m = -\frac{a}{b}}$ — ①

② Line ∥ to x-axis & y-axis



$y=c$ $x=c$

c = constant.

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③ If two points given, Let $A(x_1,y_1)$ & $B(x_2,y_2)$

then slope of AB, $\boxed{m = \frac{y_2-y_1}{x_2-x_1}}$ — ②

④ for three pts A, B, C

Let m_1 = slope of AB

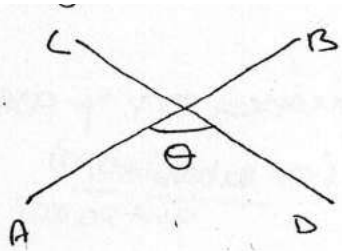
m_2 = slope of BC

then if $\boxed{m_1 = m_2}$ → ~~AB ∥ BC~~ A, B, C are collinear

if $\boxed{m_1 \cdot m_2 = -1}$ → AB ⊥ BC

⑤ if slopes of two lines is given

then they are parallel if $\boxed{m_1 = m_2}$



$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

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* System of lines:-

$a_1x + b_1y + c_1 = 0$ $\quad$ $a_2x + b_2y + c_2 = 0$

(i) Lines are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ → No solution

or $m_1 = m_2$ $\quad \left[-\frac{a_1}{b_1} = -\frac{a_2}{b_2}\right]$

(ii) Lines are ⊥r if $m_1 \cdot m_2 = -1$

or $\left(-\frac{a_1}{b_1}\right)\left(\frac{a_2}{b_2}\right) = 1 \rightarrow \frac{a_1}{b_1} = -\frac{b_2}{a_2} \rightarrow a_1a_2 = -b_1$

$a_1a_2 + b_1b_2 = 0$

(iii) Lines are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(Collinearity)

---

*(1) Length of ⊥r or distance of a point (x_1, y_1) from a line $(ax+by+c=0)$

$$= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$$

(2) if point is origin $(0,0)$ → $\left|\frac{c}{\sqrt{a^2+b^2}}\right|$

(3) Distance b/w two ∥ lines

$ax + by + c_1 = 0$ → $\left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$

Q1) ... $2\sqrt{17}$

① Find the coordinates a point situated on y-axis at a distance of 7 units above x-axis? (or below $\rightarrow (0,-7)$ and so on.) $(0,7)$

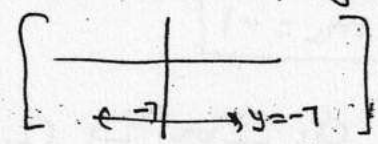
② Find distance of point $(6,-8)$ from origin? (10 units)

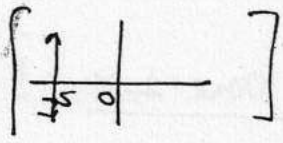
③ The line $4x+7y=12$ meets x-axis at the point:- $(3,0)$

④ Find slope of line $3x+7y+8=0$? $[y=mx+c]$ $\left(-\frac{3}{7}\right)$

⑤ Find slope of line joining $P(-4,7)$ and $Q(4,3)$? $\left[\frac{y_2-y_1}{x_2-x_1}\right] \rightarrow \left(-\frac{2}{3}\right)$

⑥ Find the eq^n of line parallel to y-axis at a distance of 5 units to the left of y-axis. ? $(x=-5)$





⑦ — — — Parallel to x-axis at a distance of 7 units below x-axis? $(y=-7)$

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⑧ Find the area of a Δ whose vertices are $P(4,5)$, $Q(-3,8)$ and $R(3,-4)$? $\left[\Delta = \frac{1}{2}\left|4(4+8)-3(-4-5)+3(5-8)\right| = 33 \text{ units}^2\right]$ $\left[33.5 \text{ units}^2\right]$

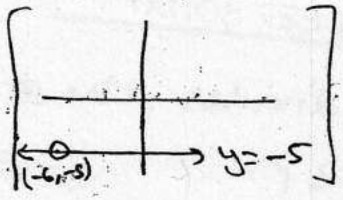
⑨ Find the coordinates of centroid of Δ with vertices $P(-2,0)$, $Q(9,-3)$ and $R(8,3)$? $(5,0)$

⑩

(11) if ... then $y^2 = ?$ $\left[(x-a)^2 + y^2 = (a+x)^2 \rightarrow y^2 = 4ax\right]$ $(4ax)$

(12) if $P(-4,a)$ and $Q(2,a+4)$ are two pts and coordinates of their middle pt are $(-1,4)$. $a = ?$ $\left[\frac{a+a+4}{2} = 4\right]$ (2)

(13) if pts $P(2,3)$, $Q(5,a)$ and $R(6,7)$ are collinear then $a = ?$ (6)
$\left[m_1 = m_2 \rightarrow \frac{a-3}{5-2} = \frac{7-3}{6-2} \rightarrow a = 6\right]$

(14) Find eqn of line parallel to x-axis and passing through $(-6,-5)$? $(y = -5)$


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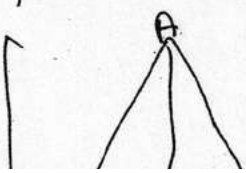
(15) Two vertices of a Δ are $P(-1,0)$ and $Q(5,-2)$ and its centroid is $(4,0)$. Find coordinates of third pt? $(8,2)$

(16) Find coordinates of pt of intersection (centroid) of medians of a Δ with vertices $P(0,6)$, $Q(5,3)$ and $R(7,3)$? $(4,4)$

(17) Find coordinates of pt which divides the line joining pts $(2,3)$ & $(7,8)$ in ratio 2:3? $\left[\overset{2}{\bullet} \quad \overset{(x,y)}{\bullet} \quad \overset{3}{\bullet}\right]$ $(4,5)$

(18) if three pts of a Δ are $(4,6)$, $(-3,-2)$ and $(-2,3)$ then find the length of median passing through A?


$AD = \sqrt{\left[4-\left(-\frac{5}{2}\right)\right]^2 + \left(6-\frac{1}{2}\right)^2}$ $\left(\frac{\sqrt{290}}{2}\right)$

A(3,4) and B(−5,−4)? [m:n, (3,3), 2 = (−5m+3n)/(m+n)] (1:7)

(20) The ratio in which the line segment joining P(−3,7) and Q(7,5) is divided by y-axis is? [~~~] (3:7)

[at y-axis → x=0 so (7m−3n)/(m+n) = 0]

(21) Value of P for which lines 3x+8y+9=0 and 24x+Py+19=0 are perpendicular is? (−9)

[m1·m2 = −1 → (−3/8)(−24/P) = −1 → P = −9]

(22) Find co-ordinates of point P which divides the join of A(3,−2) and B(1/2, 21/2) in ratio 2:3? (4,3)

(23) if P(3,5), Q(4,5) and R(4,6) be any three pts, then find angle b/w PQ & PR? [m1 = (5−5)/(4−3) = 0, m2 = (6−5)/(4−3) = 1 ; tanθ = |(m1−m2)/(1+m1m2)| = 1, θ = 45] (45°)

(24) if x cosθ + y sinθ = 2 is ⊥r to line x−y=3 then θ=? (45°)

[m1·m2 = −1 → m1 = −cotθ, m2 = 1
so (−cotθ)(1) = −1 → cotθ = 1 → θ = 45°]

$2x+7$ → is a polynomial of degree 1. (in x)

$3y^2-7y+5$ is a polynomial of degree 2. (in y)

$3z^3-\frac{7}{2}z^2+6\sqrt{5}z+\sqrt{7}$ is a polynomial in z of degree 3.

$\frac{1}{x+3}$, $\frac{5}{x^2-3x+1}$, $\sqrt{x}+5$ are not polynomials.

NOTE:- For polynomials degree of x should be a positive integer.

*Types of Polynomials :- Linear — degree 1
Quadratic — " 2
Cubic — " 3
Biquadratic — " 4

* Zeros (Roots) of a polynomial (α, β)

→ (1) ax^2+bx+c → quadratic.

$\boxed{\alpha+\beta=\frac{-b}{a}}$, $\boxed{\alpha.\beta=\frac{c}{a}}$

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STD quad. eqn → $x^2-(\alpha+\beta)x+\alpha.\beta$

→ (2) ax^3+bx^2+cx+d → cubic

$\alpha+\beta+\gamma=\frac{-b}{a}$ $\quad(\alpha\beta+\beta\gamma+\gamma\alpha)=\frac{c}{a}$

$\alpha.\beta.\gamma=\frac{-d}{a}$

STD cubic eqn → $x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha.\beta.\gamma$

if $(x-a)$ is a factor of a polynomial $f(x)$ then

$f(a)=0$

→ if $x+a$ is a factor then $f(-a)=0$

→ if $ax-b$ is a factor then $f\left(\frac{b}{a}\right)=0$ ~~so on.~~

→ if $(x-a)$ $(x-b)$ are factors then $f(a)=0$ & $f(b)=0$.

* Remainder theorem

if a polynomial $f(x)$ is divided by $(x-a)$ then remainder will be $f(a)$ i.e. put $x=a$ in polynomial.

* HCF & LCM of Polynomials

HCF → HCF is a polynomial of highest degree dividing each one of the given polynomials.

eg → $(x+3)^2(x-2)^3$, $(x-1)(x+3)(x-2)^2$

H.C.F. → $(x+3)(x-2)^2$

LCM → LCM is a polynomial of smallest degree which is divided by each one of the given polynomials.

eg → $(x-3)(x+4)^2$ and $(x-3)^3(x+4)(x+2)$

L.C.M. → $(x-3)^3(x+4)^2(x+2)$

① if ... ? (-1)

② if $(x-a)$ is a factor of $(x^3-3x^2a^3+2a^2x+b)$ then $b=?$

$[f(a)=0 \Rightarrow a^3-3a^3+2a^3+b=0]$ (0)

③ if $x^{100}+2x^{99}+k$ is div. by $(x+1)$ then $k=?$ (1)

④ if common factor of x^2+bx+c and x^2+mx+n is $(x+a)$ then $a=?$ $\left(\frac{c-n}{b-m}\right)$

$\left[\begin{array}{l} f(-a)=0 \rightarrow a^2-ba+c=0 \\ g(-a)=0 \rightarrow a^2-ma+n=0 \end{array}\right]$ equate

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⑤ if $(x^{11}+1)$ is divided by $(x+1)$ then remainder? (0)

$[x=-1 \rightarrow (-1)^{11}+1 \rightarrow -1+1=0]$

⑥ if $x^4-3x^3+2x^2-5x+7$ is divided by $(x-2)$, rem = ? (-3)

[put $x=2$]

⑦ if $(x+1)$ and $(x-2)$ be the factors of $x^3+(a+1)x^2-(b-2)x-6$ then find a & b? $(1,7)$

[put $x=-1$ and $x=2$ & solve]

⑧ when (x^3-2x^2+px-q) is divided by (x^2-2x-3), then rem. is $(x-6)$. Find p & q? $(-2,6)$

⑨ if $x+a$ & $x+a$ are divided by $(x-2)$, the same rem. are obtained. Find a ? (-3)

$f(2) = 17+4a$
$g(2) = 8+a$
so $17+4a = 8+a$
$a = -3$

⑩ if the polynomial $x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $(x-1)$ and $(x+1)$, the rem. are 5 & 19 resp. Find a, b? (5, 8)

$x=1 \rightarrow 1-2+3-a+b = 5 \Rightarrow b-a = 3$ — ①
$x=-1 \rightarrow 1+2+3+a+b = 19 \quad b+a = 13$ — ②

---

⑪ if α & β are zeros of x^2+3x+7 then find $\alpha+\beta$ = ? (-3)

$\alpha+\beta = \frac{-b}{a} \rightarrow \frac{-3}{1} \rightarrow -3$

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⑫ if α & β are zeros of $2x^2+3x-10$ then find $\alpha \cdot \beta$ = ? (-5)

$\alpha \cdot \beta = \frac{c}{a} \rightarrow \frac{-10}{2} = -5$

---

⑬ factorize → $x^4 + 5x^3 + 6x^2$ = ?

$x^2(x^2+5x+6) \rightarrow x^2(x+3)(x+2)$

⑭ factorize → $x^4 + 4$ = ?

$(x^2)^2 + 2^2 + 4x^2 - 4x^2 \rightarrow (x^2+2)^2 - (2x)^2 \rightarrow (x^2+2+2x)(x^2+2-2x)$

⑮ factorize → $(x+y)^3 - (x-y)^3$ = ?

$[(x+y)-(x-y)][(x+y)^2 + (x-y)^2 + (x+y)(x-y)]$
$= 2y[2(x^2+y^2) + (x^2-y^2)]$

(16)

$(x^4)^2+(y^4)^2+2x^4y^4-x^4y^4 \rightarrow (x^4+y^4)^2-(x^2y^2)^2$

$(x^4+y^4+x^2y^2)(x^4+y^4-x^2y^2)$

$[(x^2+y^2)^2-(xy)^2][x^4+y^4-x^2y^2]$

$\boxed{(x^2+y^2+xy)(x^2+y^2-xy)(x^4+y^4-x^2y^2)}$

(17) Factorize $\rightarrow x^4+x^2+25=?$

$(x^2)^2+(5)^2+10x^2-9x^2 \rightarrow (x^2+5)^2-(3x)^2$

$\boxed{(x^2+5+3x)(x^2+5-3x)}$

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(18) Find HCF of (x^3+x^2+x+1) and (x^4-1) ?

① $\rightarrow x^2(x+1)+1(x+1) \rightarrow (x^2+1)(x+1)$

② $\rightarrow (x^2+1)(x^2-1) \rightarrow (x^2+1)(x+1)(x-1)$

$\rightarrow$ HCF $= (x^2+1)(x+1)$

---

(19) Find HCF of (x^2-4), (x^2-5x-6) and (x^2+x-6) ?

① $\rightarrow (x+2)(x-2)$

② $\rightarrow (x-6)(x+1)$

③ $\rightarrow (x+3)(x-2)$

HCF = 1 (no common factor)

---

(20) Find HCF of $2(x^2-y^2)$ and $5(x^3-y^3)$?

① $\rightarrow 2(x+y)(x-y)$

② $\rightarrow 5(x-y)(x^2+y^2+xy)$

HCF $= (x-y)$

---

(21) f ... -4) ?

[④ LCM → $(x+1)^2(x+3)^2(x-2)(x+4)$]

(22) Find LCM of $(x+2)^2(x-2)$ and $(x^2-4x-12)$?

[② → $x^2-4x-12 \to (x-6)(x+2)$
LCM → $(x+2)^2(x-2)(x-6)$]

(23) Find LCM of (a^3+b^3) and (a^4-b^4) ?

[① → $(a+b)(a^2+b^2-ab)$
② → $(a^2-b^2)(a^2+b^2) \to (a-b)(a+b)(a^2+b^2)$
LCM → $(a+b)(a-b)(a^2+b^2-ab)(a^2+b^2)$]

(24) What should be subtracted from $27x^3-9x^2-6x-5$ to make it exactly div. by $(3x-1)$? (-7)

[Put $x=\frac{1}{3}$ in eqn → Rem → -7]

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* IMP RESULTS :-

(1) (x^n-a^n) is divided by $(x-a)$ for all values of n

(2) (x^n-a^n) is div. by $(x+a)$ for all even values of n

(3) (x^n+a^n) is div. by $(x+a)$ for all odd values of n.

(4) (x^n+a^n) is never divisible by $(x-a)$

$(a+b)^2 \rightarrow a^2+b^2+2ab$ $\quad (a-b)^2 \rightarrow a^2+b^2-2ab$

$(a+b)^2+(a-b)^2 \rightarrow 2(a^2+b^2)$

$(a+b)^2-(a-b)^2 \rightarrow 4ab$

$a^4-b^4 \rightarrow (a^2+b^2)(a+b)(a-b)$

$(a+b+c)^2 \rightarrow a^2+b^2+c^2+2(ab+bc+ca)$

$(a+b)^3 \rightarrow a^3+b^3+3ab(a+b)$

$(a-b)^3 \rightarrow a^3-b^3-3ab(a-b)$

$a^3+b^3 \rightarrow (a+b)(a^2+b^2-ab)$

$a^3-b^3 \rightarrow (a-b)(a^2+b^2+ab)$

$\rightarrow a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= \frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

if $a+b+c=0$ then $\underline{a^3+b^3+c^3=3abc}$

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if $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

C&D Rule

if $\frac{a+b}{a-b}=\frac{c}{d}$ then $\frac{a}{b}=\frac{c+d}{c-d}$

or $x^2+y^2+z^2=0$ then $x=y=z=0$. [only for squares]

T2 → if $x+\frac{1}{x}=2$ then Put $x=1$ & solve.

T3 → if $x+\frac{1}{x}=-2$ then Put $x=-1$ & solve.

T4 → if $x+\frac{1}{x}=a$

then $\left[x^2+\frac{1}{x^2}=a^2-2\right]=b$ —(1)

$\left[x^4+\frac{1}{x^4}=b^2-2\right]$ —(2) where $b=a^2-2$.

eg → $x+\frac{1}{x}=3 \rightarrow x^2+\frac{1}{x^2}=7 \rightarrow x^4+\frac{1}{x^4}=49-2=47$.

T5 → if $x-\frac{1}{x}=a$ then

(1) → $\left[x^2+\frac{1}{x^2}=a^2+2\right]=b$ and $\left[x^4+\frac{1}{x^4}=b^2-2\right]$ (2) where $b=a^2+2$.

eg → $x-\frac{1}{x}=3 \rightarrow x^2+\frac{1}{x^2}=9+2=11 \rightarrow x^4+\frac{1}{x^4}=11^2-2=119$.

T6 → if $x^4+\frac{1}{x^4}=a$ then

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(I) $\left[x^2+\frac{1}{x^2}=\sqrt{a+2}\right]=b$

(II) $\left[x+\frac{1}{x}=\sqrt{b+2}\right]$

(III) $\left[x-\frac{1}{x}=\sqrt{b-2}\right]$

eg → $x^4+\frac{1}{x^4}=119 \rightarrow x^2+\frac{1}{x^2}=\sqrt{119+2}=11 \rightarrow x+\frac{1}{x}=\sqrt{11+2}$

T7

where $x+\frac{1}{x}=k$

$$x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right) \Rightarrow \boxed{k^3+3k}$$

where $x-\frac{1}{x}=k$

T8 if $x+\frac{1}{x}=1$ then put $x^3=-1$ & solve.

if $x+\frac{1}{x}=-1$ then put $x^3=1$ & solve.

T9 if $x+\frac{1}{x}=\sqrt{3}$ then $x^3+\frac{1}{x^3}=0$

and $x^6=-1$

T10 if $x^2+\frac{1}{x^2}=1$, then $\boxed{x^6=-1}$

T11 if $x^2+\frac{1}{x^2}=-1$, then $\boxed{x^6=1}$

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① $[a=2, b=-3, c=4 \Rightarrow 2-(-3)+4 = (9)]$

② if $x^2+y^2+4x+4y+8=0$ then $x+y=?$

→ $[(x^2+4x+4)+(y^2+4y+4)=0$
$(x+2)^2+(y+2)^2=0 \Rightarrow x=-2, y=-2 \Rightarrow -2-2=(-4)]$

③ if $x+\frac{1}{x}=-2$ then $x^{32}+\frac{1}{x^{67}}=?$

$[x=-1 \rightarrow (-1)^{32}+\frac{1}{(-1)^{67}} \rightarrow 1-1=0]$

④ $x^{19}+\frac{1}{x^{23}}=2$ then $x^{23}+\frac{1}{x^{23}}=?$

$[x=1 \rightarrow 1+\frac{1}{1}=(2)]$

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⑤ $x+\frac{1}{x}=1$ then $x^{12}+x^9+x^3+1=?$

$[x^3=-1 \Rightarrow (x^3)^4+(x^3)^3+x^3+1 \Rightarrow (-1)^4+(-1)^3+(-1)+1=0]$

⑥ if $x+\frac{1}{x}=3$ then $x^4+\frac{1}{x^4}=?$

$[x^2+\frac{1}{x^2}=3^2-2=7, \; x^4+\frac{1}{x^4}=7^2-2=(47)]$

⑦ $x^4+\frac{1}{x^4}=194$ then find $x+\frac{1}{x}$ and $x^3+\frac{1}{x^3}=?$

$[x^2+\frac{1}{x^2}=\sqrt{194+2}=14 \Rightarrow x+\frac{1}{x}=\sqrt{14+2}=(4)$
$x^3+\frac{1}{x^3}=4^3-3\times4=(52)]$

⑧ $(x+\frac{1}{x})^2=3$ then $x^{72}+x^{66}+x^{54}+x^{36}+x^{24}+x^6+1=?$

$[x+\frac{1}{x}=\sqrt{3} \Rightarrow x^6=-1 \rightarrow (x^6)^{12}+(x^6)^{11}+(x^6)^9+(x^6)^6+(x^6)^4+x^6+1$

(8) $x^4 \quad x \quad x^3$

$x^2+\frac{1}{x^2}=\sqrt{119+2}=11 \Rightarrow x+\frac{1}{x}=\sqrt{11+2}$ but $x-\frac{1}{x}=\sqrt{11-2}=3$

$x^3-\frac{1}{x^3}=k^3+3k=3^3+3\times3=36$

(9) $\frac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}=\frac{3}{2}$ then $x=?$

$\frac{a+b}{a-b}=\frac{c}{d}$ so $\frac{a}{b}=\frac{c+d}{c-d} \Rightarrow \frac{\sqrt{x+2}}{\sqrt{x-2}}=\frac{3+2}{3-2} \Rightarrow \frac{x+2}{x-2}=\frac{25}{1}$,

$x+2=25x-50 \Rightarrow x=\frac{52}{24}=\frac{13}{6}$

(10) if $4x-\frac{2}{3x}=4$ then $8x^3-\frac{1}{27x^3}=?$

$2x-\frac{1}{3x}=2 \rightarrow \left(2x-\frac{1}{3x}\right)^3 \Rightarrow 8x^3-\frac{1}{27x^3}-\cancel{3}\times2\cancel{x}\times\frac{1}{\cancel{3x}}\left(2x-\frac{1}{3x}\right)=8$

$8x^3-\frac{1}{27x^3}=8+2\times2=12$

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(11) if $\frac{x}{y}+\frac{y}{x}=-1$ then $x^3-y^3=?$

$\frac{x^2+y^2}{xy}=-1 \Rightarrow x^2+y^2+xy=0 \Rightarrow x^3-y^3=(x-y)(\underbrace{x^2+y^2+xy}_{0})=0$

(12) if $x+y=2z$ then $\frac{x}{x-z}+\frac{z}{y-z}=?$

$x-z=z-y \Rightarrow \frac{x}{x-z}-\frac{z}{z-y} \Rightarrow \frac{x}{x-z}-\frac{z}{x-z} \rightarrow \frac{x-z}{x-z}=1$

Or Put $x=1$, $y=3$, $z=\frac{1+3}{2}=2$ & solve

(13) if $2x+\frac{1}{3x}=5$ then $\frac{5x}{6x^2+20x+1}=?$

$\frac{6x^2+1}{3x}=5 \Rightarrow 6x^2+1=15x \Rightarrow \frac{5x}{15x+20x}=\frac{1}{7}$

(14) [Divide by $n \rightarrow n-3+\frac{1}{n}=0 \Rightarrow n+\frac{1}{n}=3$

$n^3+\frac{1}{n^3} = K^3-3K = 3^3-3\times3 =$ (18)]

(15) if $a^2=b+c$, $b^2=c+a$, $c^2=a+b$ then $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=?$

[$\frac{a}{a+a^2}+\frac{b}{b+b^2}+\frac{c}{c+c^2} \Rightarrow \frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}=\frac{a+b+c}{a+b+c}=1$

or Put $a=2, b=2, c=2$ & solve]

(16) if $x-y=\frac{x+y}{7}=\frac{xy}{4}$ then $xy=?$

[$x-y=\frac{x+y}{7}=\frac{xy}{4}=K \Rightarrow x-y=K$, $x+y=7K$

$x=4K, y=3K$

Also $xy=4K$

& $xy=4K\times3K=12K^2$

So $12K^2=4K \rightarrow K=\frac{1}{3}$

$xy=4K=\frac{4}{3}$]

(17) if $x+y+z=0$ then $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=?$

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[$\frac{x^3+y^3+z^3}{xyz} \Rightarrow \frac{3xyz}{xyz}=3$] or Put $x=1, y=2, z=-3$

(18) if $2^x=3^y=6^z$ then $\frac{1}{x}+\frac{1}{y}-\frac{1}{z}=?$

[$2=K^{1/x}$

$3=K^{1/y}$ $\rightarrow$ $2\times3=6$

$6=K^{1/z}$

$K^{1/x}\cdot K^{1/y}=K^{1/z} \Rightarrow K^{\frac{1}{x}+\frac{1}{y}}=K^{\frac{1}{z}} \Rightarrow \frac{1}{x}+\frac{1}{y}-\frac{1}{z}=0$]

(19) if $a^2+b^2+c^2=2(a-b-c)-3$ then $2a-3b+4c=?$

[$a^2-2a+1+b^2+2b+1+c^2+2c+1=0 \Rightarrow (a-1)^2+(b+1)^2+(c+1)^2=0$

So $a=1, b=-1, c=-1$ so $2(1)-3(-1)+4(-1) \rightarrow 2+3-4=1$]

(20) $\sqrt[3]{p^3+3p^2+3p+1} \Rightarrow \sqrt[3]{(p+1)^3} \rightarrow p+1 = 125$

(21) if $\frac{a}{b} = \frac{c}{d} + \frac{e}{f} = \frac{3}{1}$ then $\frac{2a^2+3c^2+4e^2}{2b^2+3d^2+4f^2} = ?$

Put $a=c=e=3$ & $b=d=f=1$ $\Rightarrow$ (9)

(22) if $a=25, b=15, c=-10$ then $\frac{a^3+b^3+c^3-3abc}{(a-b)^2+(b-c)^2+(c-a)^2} = ?$

$\frac{1}{2}\frac{(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]}{(a-b)^2+(b-c)^2+(c-a)^2} = \frac{a+b+c}{2} = \frac{25+15-10}{2} = 15$

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(23) if $x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}=4$ then $x^2+y^2=?$

$x^2+\frac{1}{x^2}-2+y^2+\frac{1}{y^2}-2=0 \Rightarrow (x-\frac{1}{x})^2+(y-\frac{1}{y})^2=0$

$x-\frac{1}{x}=0 \rightarrow x^2-1=0, x=1$ so $y=1 \Rightarrow x^2+y^2=1^2+1^2=2$

or Put $x=1, y=1 \rightarrow 1+1+\frac{1}{1}+\frac{1}{1}=4 \rightarrow 1^2+1^2=2$

(24) if $3^{2x-y} = 3^{x+y} = \sqrt{27}$ then $y=?$

$3^{2x-y} = 3^{3/2}$, $3^{x+y} = 3^{3/2} \Rightarrow 2x-y=\frac{3}{2}$, $x+y=\frac{3}{2} \rightarrow x=1, y=\frac{1}{2}$

(25) if $x+\frac{1}{x}=3$ then find $x^5+\frac{1}{x^5}, x^6+\frac{1}{x^6}, x^7+\frac{1}{x^7}=?$

$x^2+\frac{1}{x^2}=3^2-2=7$, $x^4+\frac{1}{x^4}=7^2-2=47$, $x^3+\frac{1}{x^3}=3^3-3\times3=18$

$(x^2+\frac{1}{x^2})(x^3+\frac{1}{x^3})=7\times18=126$

$x^5+(\frac{1}{x}+x)+\frac{1}{x^5}=126 \Rightarrow x^5+\frac{1}{x^5}=126-3=123$

x^3) x^6 x^3

$x^6+\frac{1}{x^6} = 324-2 = \boxed{322}$

$\left(x^3+\frac{1}{x^3}\right)\left(x^4+\frac{1}{x^4}\right) = 18\times 47 \Rightarrow x^7+\left(\frac{1}{x}+x\right)+\frac{1}{x^7} = 846$ (3)

$x^7+\frac{1}{x^7} = 846-3 = \boxed{843}$

---

(26) if $x+\frac{1}{x}=3$ then $x^2-\frac{1}{x^2}=?$

$x^2-\frac{1}{x^2} = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ ⟵ $3\times\sqrt{5} = \boxed{3\sqrt{5}}$

$x^2+\frac{1}{x^2} = 3^2-2 = 7$, $x-\frac{1}{x} = \sqrt{7-2} = \sqrt{5}$

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(27) if $x+\frac{4}{x}=4$ then $x^2+\frac{1}{x^3}=?$

$\frac{x^2+4}{x}=4 \Rightarrow x^2-4x+4=0 \Rightarrow (x-2)^2=0,\ x=2$

$x^2+\frac{1}{x^3} = 2^2+\frac{1}{2^3} = \boxed{4\frac{1}{8}}$

(28) if $x+\frac{1}{x}=5$ then $\frac{x^4+\frac{1}{x^2}}{x^2-3x+1}=?$

Divide by x → $\frac{x^3+\frac{1}{x^3}}{x-3+\frac{1}{x}} \Rightarrow \frac{5^3-3\times 5}{5-3} = \frac{110}{2} = \boxed{55}$

---

(29) if $5x+\frac{1}{3x}=5$ then $9x^2+\frac{1}{25x^2}=?$

$\times\frac{3}{5} \Rightarrow 3x+\frac{1}{5x}=3 \Rightarrow 9x^2+\frac{1}{25x^2}+2\cdot 3x\cdot\frac{1}{5x}=9$

$9x^2+\frac{1}{25x^2} = 9-\frac{6}{5} = \boxed{\frac{39}{5}}$

(30) $n^3+n+\frac{1}{n}+\frac{1}{n^3} \Rightarrow n+\frac{1}{n}+n^3+\frac{1}{n^3}$ —①

$n=2+\sqrt{3}$, $\frac{1}{n}=\frac{1}{2+\sqrt{3}}\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)=\frac{2-\sqrt{3}}{1}$ so $n+\frac{1}{n}=2+\sqrt{3}+2-\sqrt{3}=4$

$n^3+\frac{1}{n^3}=4^3-3\times4=52$

Put in ① → $n+\frac{1}{n}+n^3+\frac{1}{n^3}=4+52=\boxed{56}$

(31) if $n=2-2^{1/3}+2^{2/3}$ then $n^3-6n^2+18n+18=?$

$(n-2)^3=(2^{2/3}-2^{1/3})^3 \Rightarrow n^3-8-6n(n-2)=4-2-3\times2^{2/3}\times2^{1/3}(2^{2/3}-2^{1/3})$

$\Rightarrow n^3-8-6n^2+12n=2-6(n-2)$

$n^3-6n^2+18n=22$

$n^3-6n^2+18n+18=22+18=\boxed{40}$

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(32) Find $n^4-17n^3+17n^2-17n+17$ at $n=16$?

$n^4-16n^3-n^3+16n^2+n^2-16n-n+17$, Now $16=n$

$n^4-n^4-n^3+n^3+n^2-n^2-n+17 \Rightarrow -16+17=①$

Shortcut ⇒ Last two terms $-17n+17$

$-16n-n+17=-16+17=①$

break in two parts.

(33) Find $n^5-12n^4+12n^3-12n^2+12n-1$ at $n=11$?

$n^5-11n^4-n^4+11n^3+n^3-11n^2-n^2+11n+n-1$

$n^5-n^5-n^4+n^4+n^3-n^3-n^2+n^2+n-1$

$\Rightarrow n-1=11-1=10$

Shortcut → Last two terms ⇒ $12n-1$

$11n+n-1 \Rightarrow 11-1=(10)$

AREAS & PERIMETER

- TRIANGLES
- QUADRILATERALS
- CIRCLES

* TRIANGLES

(1) Equilateral Δ → all sides equal

(2) Isosceles Δ → Two sides equal

(3) Scalene Δ → NO two sides equal

(4) Right Angled Δ → one angle 90°

(5) obtuse angled Δ → one angle >90°

(6) Acute angled Δ → each angle <90°

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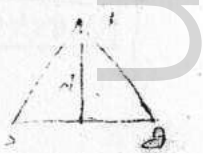
* Any side of the Δ is less than the sum of the two other sides.

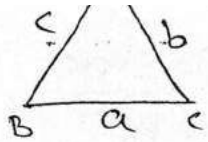
$$\left.\begin{aligned} AB &< AC+BC \\ AC &< BC+AB \\ BC &< AB+AC \end{aligned}\right] \text{ or } \left[\begin{aligned} AB+AC &> BC \\ BC+AB &> AC \\ AC+BC &> AB \end{aligned}\right.$$

* The length of a side of a Δ is not less than the absolute value of the difference b/w the other two sides.

$$AC \geq |AB-BC|$$
$$AB \geq |AC-BC|$$
$$BC \geq |AB-AC|$$



* Perimeter $\rightarrow a+b+c$



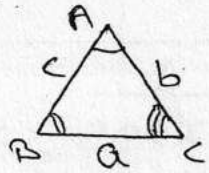
Semi-Perimeter $(s) = \frac{a+b+c}{2}$

* if we can form a Δ with lengths a, b and c then we can also form a Δ with lengths $\sqrt{a}, \sqrt{b}$ & $\sqrt{c}$.

[we can form Δ with a, b & c if $a+b>c$, $a+c>b$, $b+c>a$]

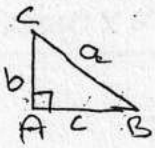
* **AREA OF Δ :-**

3 **Case (1) :-** if any two sides and angles b/w two sides of a Δ are given,



$$\text{Area of } \Delta = \frac{1}{2} bc \cdot \sin A = \frac{1}{2} ab \cdot \sin C = \frac{1}{2} ac \cdot \sin B$$

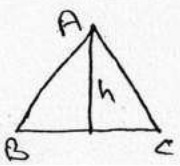
Max. Area at $90^\circ \rightarrow \frac{1}{2} bc \sin 90^\circ = \frac{1}{2} bc \cdot 1$



1 **Case 2 :-** if lengths of all three sides are given;

$$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{a+b+c}{2}$$

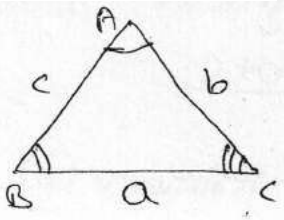
2 **Case 3 :-** if any one side and corr. altitude is given,



$$\text{Area of } \Delta = \frac{1}{2} \times B \times h$$

Case 4 (Given)

$$\text{Area of } \Delta = \frac{a^2 \sin B \sin C}{2 \sin A} = \frac{b^2 \sin A \sin C}{2 \sin B} = \frac{c^2 \sin A \sin B}{2 \sin C}$$

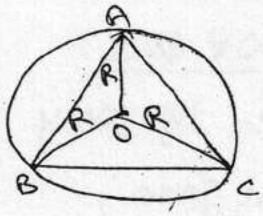


* **Sine Rule:-** In any ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

where R = circumradius of the Δ

circumcircle



$OA = OB = OC = R$

$$R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C}$$

or

$$R = \frac{abc}{4 \times \text{area of } \Delta}$$

NOTE:-
$a : b : c = \sin A : \sin B : \sin C$

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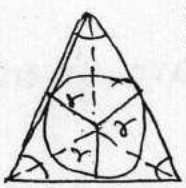
* **Incircle**



r = Inradius.

$$r = \frac{\text{Area of } \Delta}{\text{Semi-perimeter}} = \frac{\Delta}{s}$$

or

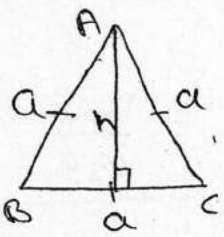
$$r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$$

* **Relation b/w circum radius & inradius:-**

$r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

* TYP

(1) EQUILATERAL Δ



$\text{Area} = \frac{\sqrt{3}}{4} a^2$

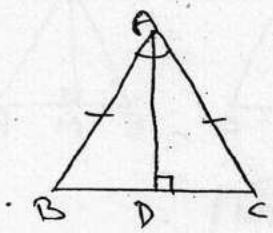
$P = a+a+a = 3a$

$h = \frac{\sqrt{3}}{2} a$

$\text{Inradius} = \frac{1}{3} \times h = \left[\frac{1}{3} \times \frac{\sqrt{3}}{2} a = \frac{a}{2\sqrt{3}}\right]$

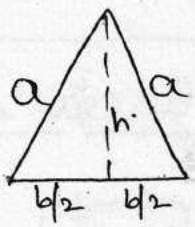
$\text{circumradius} = \frac{2}{3} \times h = \left[\frac{2}{3} \times \frac{\sqrt{3}}{2} a = \frac{a}{\sqrt{3}}\right]$

(2) Isosceles Δ (AB = AC)



AD is the angle bisector of ∠BAC, also the the ⊥r bisector of base and also the median.

[BD = DC, ∠BAD = ∠DAC, ∠ADC = 90°]



$\text{Area} = \frac{1}{2} \times b \times h = \frac{1}{2} \times b \sqrt{a^2 - \frac{b^2}{4}} = \frac{b}{2} \sqrt{\frac{4a^2 - b^2}{4}}$

$\boxed{\text{Area} = \frac{b}{4} \sqrt{4a^2 - b^2}}$

$\boxed{h = \frac{1}{2} \sqrt{4a^2 - b^2}}$

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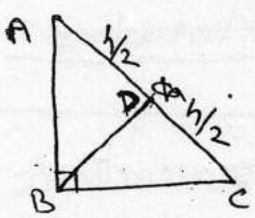
(3) Right Angled Δ

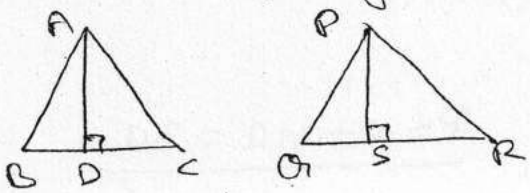


$\text{Area} = \frac{1}{2} \times AB \times BC$

if BD = median then $\boxed{BD = \frac{1}{2} \times h}$

Also BD = circumradius

(1) to their corresponding bases.



if $AD = PS$

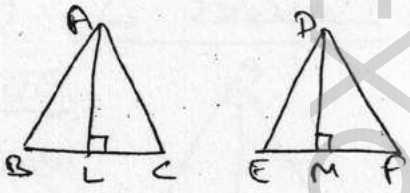
then $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{BC}{QR}$

Or if $BC = QR$ then

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AD}{PS}$$

(2) if Area of ΔABC = Area of ΔDEF and $BC = EF$

then $AL = DM$ and vice-versa.



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* Inradius of an equilateral Δ

$$r = \frac{a}{2\sqrt{3}}$$

* Circum radius of an equi. Δ

$$R = \frac{a}{\sqrt{3}}$$

$$r = \frac{R}{2}$$

* Area of a Regular Polygon $= \frac{1}{2} \times n \times r$

$= \frac{1}{2} \times$ Perimeter $\times$ inradius

~~n → no. of sides~~

~~r → inradius~~

* Note!- The largest Δ that can be inscribed in a semicircle is right angled isosceles Δ. $A = r^2$ $\left[\frac{1}{2} \times 2r \times r\right]$

* POLYGONS

Sum of all the interior angles of a regular Polygon = $(n-2) \times 180^\circ$

Sum of all the exterior angles = 360°

Perimeter = na

Area of the regular Polygon = $\frac{na^2}{4} \cot\left(\frac{\pi}{n}\right)$

$= \left[\frac{1}{2} (\text{Perimeter}) \times \text{Ir from centre to any side}\right]$

[Area of square = $4 \frac{a^2}{4} \cot \frac{\pi}{4} = a^2 \cdot \cot 45^\circ = a^2$]

Area of regular hexagon = $\frac{6a^2}{4} \cdot \cot \frac{\pi}{6} = \frac{3a^2}{2} \cdot \cot 30^\circ$

$= \frac{3a^2}{2} (\sqrt{3}) = \boxed{\frac{3\sqrt{3}}{2} a^2}$

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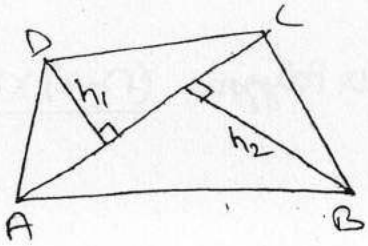
* Area enclosed b/w circumcircle and incircle of any regular polygon of side 'a' is $\boxed{\frac{\pi a^2}{4}}$

* Qu



Area of Quad. = Area of $\triangle ADC$ + Area of $\triangle ABC$

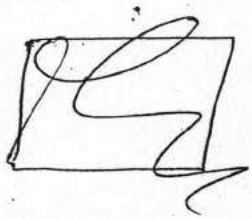
$= \frac{1}{2} d h_1 + \frac{1}{2} d h_2$

$= \boxed{\frac{1}{2} d (h_1 + h_2)}$

Area of Quad. = $\frac{1}{2}$ x diagonal x sum of ⊥rs to it from opp. vertices.

* Types of Quadrilaterals

(1) Parallelogram
(2) Rectangle
(3) Square
(4) Rhombus
(5) Trapezium
(6) cyclic quad.



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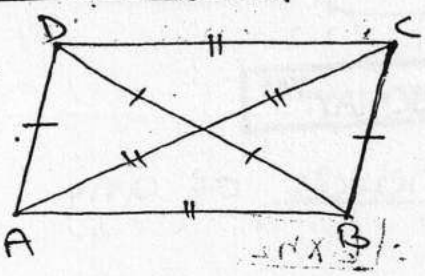
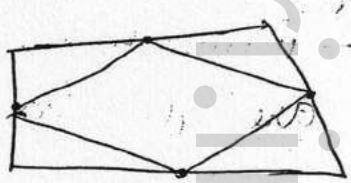
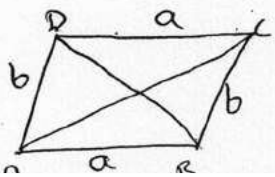
(1) Parallelogram



- AB ∥ CD & AB = CD
- BC ∥ DA & BC = DA
- Diagonals AC and BD bisect each other.
- opposite angles are equal
- sum of two adjacent angles is 180°
- each diagonal divides ∥gm into two $\triangle$s of equal areas.
- Straight line joining the midpts of adjacent sides of any quad. form a ∥gm.

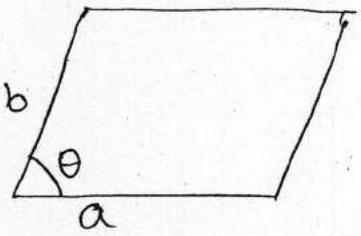




$\boxed{d_1^2 + d_2^2 = 2(a^2 + b^2)}$

Area of the ll gm :-

Case 1:- if the lengths of two adjacent sides and the angle blw them is given.

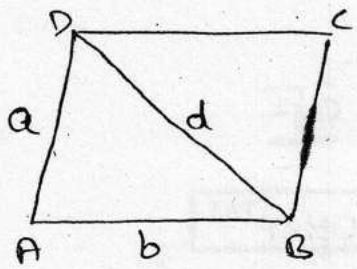


$A = ab \sin\theta$

$\sin\theta$ = max.(1) at $\theta = 90°$

So Rectangle is the ll gm having largest possible area. = ab

Case 2:- if lengths of two adjacent sides and one diagonal is given.



$$\text{Area} = 2\sqrt{s(s-a)(s-b)(s-d)}$$

where $s = \frac{a+b+d}{2}$

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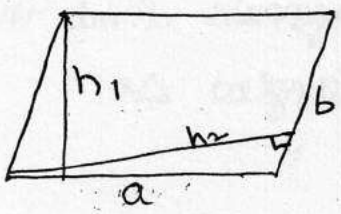
Case 3:- if the length of one of the sides, is given and ⊥r distance from opp. side is given.



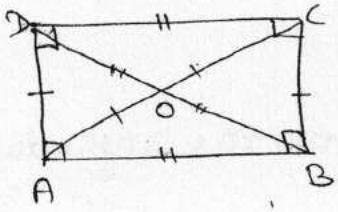
$\text{Area} = \text{Base} \times \text{ht} = ah_1 = bh_2$

NOTE:-

→ ll gms that lie on same base and blw same ll lines are equal in area.

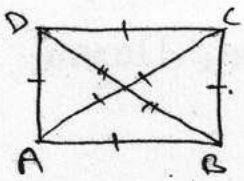
→ if a Δ and a llgm lie on same base and blw same ll lines then Area of Δ = ½ Area of llgm.

② R ... 90° angle



- Diagonals AC = BD and bisect each other
- Perimeter = $2(l+b)$
- Area = $l \times b$
- Diagonal = $\sqrt{l^2+b^2}$

③ **Square** - A ||gm having all sides equal and each angle 90°



- Diagonals AC = BD & bisect each other
- Perimeter = $4a$
- Area = $a^2 = \frac{1}{2} \times d^2$
- Diagonal = $\sqrt{a^2+a^2} = \sqrt{2a^2} = a\sqrt{2}$

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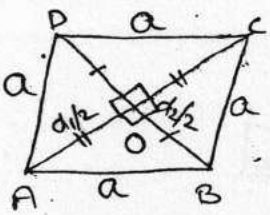
④ **Rhombus** - A ||gm in which all sides are equal but angles not of 90°.



- Diagonals not equal but bisect each other at right angles (90°) i.e. Diagonals divide the rhombus into four equal right angled Δs.
[$\Delta BOC = \Delta AOB = \Delta AOD = \Delta COD$]

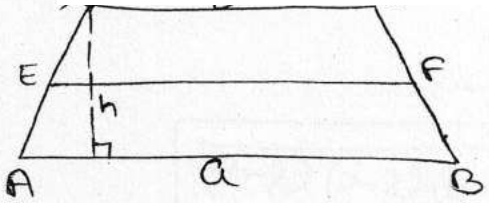
- In rt Δ COD, $OD^2 + OC^2 = DC^2$

$$\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = a^2$$

$$d_1^2 + d_2^2 = 4a^2$$

or $a = \frac{1}{2}\sqrt{d_1^2+d_2^2}$

- Area = 4 × Area of rt Δ COD = $4 \times \frac{1}{2} \times OD \times OC = 4 \times \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2}$



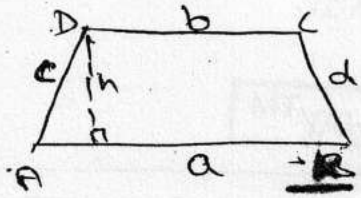
→ if EF = median (line segment joining midpts of non-|| sides)

∴ then $\boxed{EF = \frac{1}{2}(a+b)}$

Area of Trapezium

(1) $\boxed{A = \frac{1}{2}(a+b)h}$

(2) if lengths of || sides and lengths of non-|| sides are given.



$K = (a-b)$ → Diff. b/w || sides.

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- ht, $\boxed{h = \frac{2}{K}\sqrt{S(S-K)(S-c)(S-d)}}$

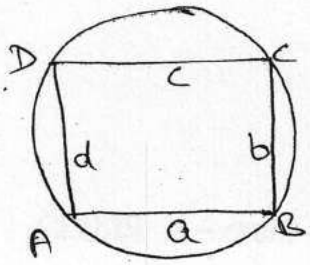
$$\text{Area} = \boxed{\frac{1}{2}(a+b)\cdot h} = \frac{1}{2}(a+b)\cdot\frac{2}{K}\sqrt{S(S-K)(S-c)(S-d)}$$

$$\boxed{\text{Area} = \frac{(a+b)}{K}\sqrt{S(S-K)(S-c)(S-d)}} \quad \cancel{\boxed{\frac{1}{2}(a+b)\cdot h}}$$

NOTE :-

→ A trapezium inscribed in a circle is isosceles ie. its non-|| sides are equal.

⑥ C



$\angle B + \angle D = \angle A + \angle C = 180°$

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where $s = \frac{a+b+c+d}{2}$

→ The quad. formed by the angle bisectors of a cyclic quad. is also cyclic.

→ if two non-|| sides of a trapezium are equal then it is cyclic and its diagonals are also equal.

$$ac + bd = d_1 \times d_2$$

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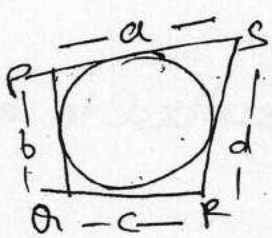
* Area of 4 walls of a room = $lh + bh + lh + bh$

$= 2lh + 2bh$

$= 2h(l+b)$

* Percentage change in Area

$$\left(\pm a \pm b + \frac{(\pm ab)}{100}\right)\%$$



PQRS → quad.

Area of circle $= \sqrt{abcd}$

$a + c = b + d$

① Find the perimeter of the ||gm whose two sides are in ratio 3:2 and area = 243 ~~sq~~ m^2. and angle blw two sides is 30°. ($90m^2$)

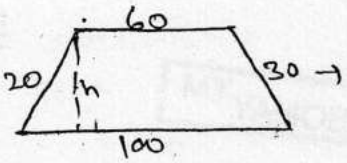
[Area = $(6x^2)\sin 30° = 243$
$3x^2 = 243 \rightarrow x = 9$, Peri. $= 2(2x+3x) = 90m^2$]

② The ratio blw the diagonals of a rhombus is 3:5 and its area is 120 cm^2. Find length of the bigger diagonal? (20cm)

[Area = $\frac{1}{2}d_1d_2 \Rightarrow \frac{1}{2}\times 15x^2 = 120 \rightarrow x = 4$ Ans. $= 5\times 4 = 20$]

③ In a trapezium, the ratio of lengths of || sides is 2:3 and the ht. of trapezium is $\frac{3}{4}$ times the smaller side. Area of trapezium is 60 cm^2. Find length of smaller || side? (8 cm)

[Area = $\frac{1}{2}(2x+3x)\cdot\frac{3}{4}\cdot 2x \Rightarrow x = 4$ so Ans $\rightarrow 2\times 4 = 8$]

④ 

Find Area? ($300\sqrt{15}$ cm^2)

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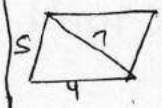
[$h = \frac{2}{K}\sqrt{S(S-K)(S-c)(S-d)} = \frac{2}{40}\sqrt{45\times 5\times 15\times 15} = \frac{15\sqrt{15}}{4}$
$K = 100-60 = 40$
$S = \frac{40+20+30}{2} = 45$
Area $= \frac{(100+60)}{2}\cdot\frac{15\sqrt{15}}{4} = 300\sqrt{15}$ cm^2]

or [$A = \frac{(a+b)}{K}\sqrt{S(S-K)(S-c)(S-d)}$]

⑤ The ... and respective diagonal is ... cm. Find area of ||gm? (19.6 cm²)



Area = $2\sqrt{s(s-a)(s-b)(s-d)} = 2\sqrt{8\times3\times4\times1} = 8\sqrt{6} = 19.6\ cm^2$

$s = \frac{5+4+7}{2} = 8$

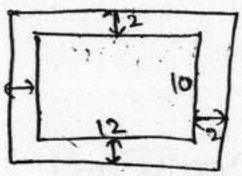
⑥ In a ||gm, the lengths of adjacent sides are 12cm & 14cm. if one diagonal is 16cm then find length of other diagonal? (20.6cm)

$d_1^2 + d_2^2 = 2(a^2+b^2) \rightarrow 16^2 + x^2 = 2(12^2+14^2) \rightarrow x^2 = 424$, $x = \sqrt{424} = 20.6cm$

⑦ Find the diagonal of a rectangle whose sides are 12m and 5m? (13m)

$[\sqrt{12^2+5^2} = 13]$

⑧ A rectangular hall 12m long, 10m broad is surrounded by a verandah 2 metres wide. Find the area of verandah? ($104\ m^2$)



$16\times14 - 12\times10$

$224 - 120 = 104m^2$

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⑨ Find the area of a rhombus whose one side is 20cm and one diagonal 24cm? ($384\ cm^2$)

$d_1^2 + d_2^2 = 4a^2 \rightarrow d_2^2 = 4(20)^2 - 24^2 = 1024 \rightarrow d_2 = 32$

Area $= \frac{1}{2}\times24\times32 = 384\ cm^2$

⑩ Find the area of regular hexagon whose side is ... (21) ($210.4\ cm^2$)

$[A = 3\sqrt{3}a^2 = 3\sqrt{3}\times...$

(11) A door $2m \times \frac{3}{2}m$. Find the cost of papering the walls with paper at 25 P per metre?

Area of walls $= 2 \times 3(6+8) = 84\ m^2$

Area of windows and doors $= 2 \times \frac{3}{2} \times 1 + 2 \times \frac{3}{2} = 6\ m^2$

Area to be covered $= 84 - 6 = 78\ m^2$

Cost $= \frac{156 \times 25}{100} = 39$ ₹

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(12) A hall whose length is 16 m and breadth is twice its ht, takes 168 m of paper, 2 m wide, for its four walls. Find the area of floor? ($192\ m^2$)

ht $= h$, breadth $= 2h$

Area of four walls $\Rightarrow 2h(16+2h) = 168 \times 2 \rightarrow h = \frac{6, -14}{\times}$

$h = 6$, breadth $= 12$ m.

Area of floor $= 16 \times 12 = 192\ m^2$

(13) if length & breadth of a rectangle is inc. by 5% and 4%, then by what % does the area of rectangle increase?

$5 + 4 + \frac{20}{100} = 9 + 0.2 = +9.2\%$

(14) if length of rectangle dec. by 4% and breadth inc. by 6%, then find % change in area?

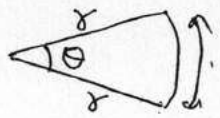
$-4 + 6 - \frac{24}{100} = 2 - 0.24 = 1.76\%$ inc.

(15) if sides of a square are inc. by 10% then change in area?

$10 + 10 + \frac{100}{100} = 21\%$ inc.

* circumference = $2\pi r$

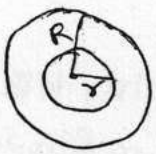
Area = $\pi r^2 = \frac{\pi D^2}{4}$

* 

~~Area of~~ length of Arc = $\frac{\theta}{360} \times 2\pi r$

Area of Arc = $\frac{\theta}{360} \times \pi r^2$

* Area enclosed by two concentric circles



$\pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

$= \pi(R+r)(R-r)$

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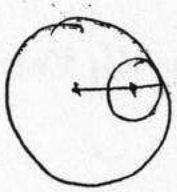
*① → if two circles touch internally, then the distance b/w their centres is equal to the difference of their radii.

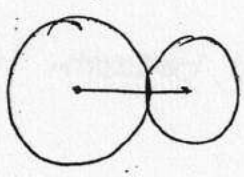
② if two circles touch externally, then the distance b/w their centres is equal to the sum of their radii.

③ Distance moved by a rotating wheel in one revolution is equal to the circumference of wheel.

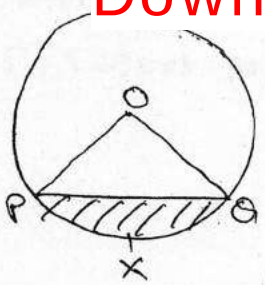
and no. of revolutions = $\frac{\text{Distance moved}}{\text{circumference}}$



①



②



Area of minor segment PXQ

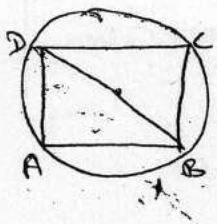
= Area of sector OPXQ − Area of Δ OPQ

Area of major segment PYQ

= Area of circle − Area of minor segment

---

(1) Area of a square inscribed in a circle of radius r is $2r^2$



Diagonal BD = $2r$ and diagonal = $\sqrt{2}a$

Area of square = $a^2 = \left(\frac{d}{\sqrt{2}}\right)^2 = \frac{4r^2}{2} = 2r^2$

and side of square = $\frac{2r}{\sqrt{2}} = \sqrt{2}r$

Square is the largest ~~circle~~ quad. inscribed in a circle

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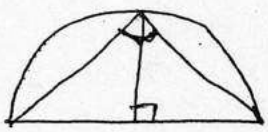
(2) The Area of the largest triangle inscribed in a semicircle of radius r is r^2.

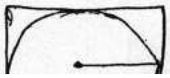


Largest Δ is isosceles Δ.

Base → diameter, ht → radius.

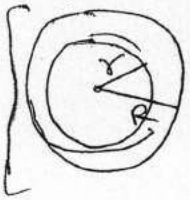
Area = $\frac{1}{2} \times \text{base} \times \text{ht} = \frac{1}{2} \times 2r \times r = r^2$

(3) The area of the largest circle that can be drawn in a square of side x is $\pi\left(\frac{x}{2}\right)^2$



radius = $\frac{x}{2}$

① A [illegible]erence is 352 m and outer circumference is 396 m. Find width of track? (7 m)



$2\pi R = 396 \rightarrow R = 63\ m$

$2\pi r = 352 \rightarrow r = 56\ m$

$R - r = 63 - 56 = 7\ m$

② The hour and minute hands of a clock are 4 cm and 6 cm long resp. find the sum of distances travelled by their tips in 2 days? (1910.57 cm)

Hr hand will complete 4 rounds in 2 days

Distance moved $= 4 \times (2 \times \frac{22}{7} \times 4) = \frac{704}{7}$ cm

min hand will complete 48 rounds in 2 days.

$= 48 \times (2 \times \frac{22}{7} \times 6) = \frac{12672}{7}$ cm $\rightarrow$ Total $= \frac{704}{7} + \frac{12672}{7} = 1910.57$ cm

③ The radius of a circular wheel is $1\frac{3}{4}$ m. How many revolutions will it make in travelling 11 km? (1000)

circumference $= 2 \times \frac{22}{7} \times \frac{7}{4} = 11\ m \rightarrow 1$ revolution

$11000\ m \rightarrow 1000$ revolutions

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④ if radius of a circle is increased by 5%, find the % age inc. in its area? (10.25%)

$\pi \times r \times r \rightarrow 5 + 5 + \frac{25}{100} \rightarrow 10.25\%$

⑤ if diameter of a circle is inc. by 12%, find the % inc. in its circumference? $[2\pi r \rightarrow 12\%]$ (12%)

⑥ A semicircle is constructed on each side of a square of length 2m. Find area of whole figure?

if th... 22 cm².

area of that circle? (28 cm²)

Perimeter = $4\sqrt{22} = 2\pi r \rightarrow r = \frac{2\sqrt{22}}{\pi}$

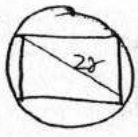
Area = $\pi r^2 = \pi\left(\frac{2\sqrt{22}}{\pi}\right)^2 = \frac{\pi \times 4 \times 22}{\pi^2} = \frac{4 \times 22 \times 7}{22} = 28\ cm^2$

---

(8) The circumference of a circle is 88 cm. Find the side of the square inscribed in the circle?

$2\pi r = 88 \rightarrow r = \frac{88 \times 7}{2 \times 22} = 14$ cm.

side of square = $\sqrt{2}r = \sqrt{2} \times 14$ cm



$\{d = 2r \times r,\ a = \frac{2r}{\sqrt{2}} = \sqrt{2}r\}$

---

(9) The largest Δ is inscribed in a semicircle of radius 14 cm. Find the area inside the semicircle which is not occupied by Δ?

Area of circle − Area of Δ

$\frac{\pi r^2}{2} - r^2 \rightarrow r^2\left(\frac{\pi}{2} - 1\right) = 112\ cm^2$

$\{\frac{1}{2} \times r \times r = r^2\}$

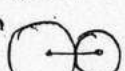
---

(10) Find the area of largest circle that can be inscribed in a square of side 14 cm?

$\pi r^2 \rightarrow \pi\left(\frac{14}{2}\right)^2 = \frac{22}{7} \times 7^2 = 154\ cm^2$

---

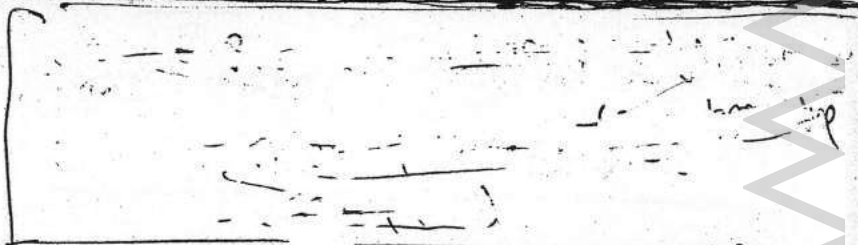
(11) Two circles touch externally, The sum of their areas is 130π cm² and distance blw their centres is 14 cm. Find radii of circles?



$\pi r_1^2 + \pi r_2^2 = 130\pi \rightarrow r_1^2 + r_2^2 = 130$

SURFACE AREAS & VOLUMES

1) CUBOID
2) CUBE
3) CYLINDER
4) CONE
5) SPHERE – HEMISPHERE
6) RIGHT PRISM
7) RIGHT PYRAMID



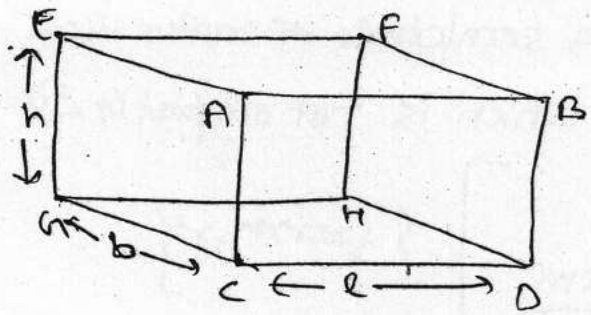
For uniform shapes (Base – Top same)

C.S.A. = Perimeter of base × ht

T.S.A. = C.S.A. + 2 × Area of base

Volume = Area of base × ht.

[1] CUBOID → A cuboid is bounded by six rectangular faces. 2 opposite faces are equal (Room)



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8 vertices → A, B, C, D, E, F, G, H

12 edges → AB, CD, AC, BD etc.

6 faces

No. of faces + No. of vertices = No. of edges + 2

SURFACE AREA = $2(\ell b + bh + \ell h)$

Volume of Cuboid = Area of any face × corresponding height (for All)

= Area of base × height

= $\ell \times b \times h$

Length of the face Diagonals = $\sqrt{\ell^2 + b^2}$ or $\sqrt{\ell^2 + h^2}$

→ if $lb = A_1$, $bh = A_2$, $lh = A_3$

then $A_1 \cdot A_2 \cdot A_3 = (lb)(bh)(lh) = l^2b^2h^2$

$$\text{Volume} = \sqrt{A_1 A_2 \cdot A_3}$$

* Area of four walls of a Room $= 2lh + 2bh = 2h(l+b)$

$= [\text{Perimeter of base} \times h]$

$= [2(l+b) \times h]$

∴ L.S.A.

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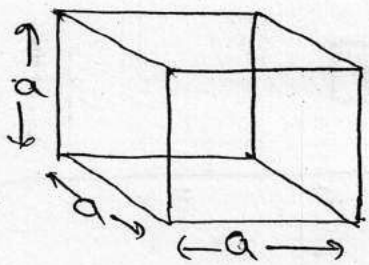
2. CUBE → A cube is bounded by six square faces ($l = b = h$)



six faces
8 vertices
12 edges

Area of each face $= a^2$

1. Total surface area $= 6a^2$

face Diagonals $= \sqrt{a^2+a^2} = \sqrt{2}a$

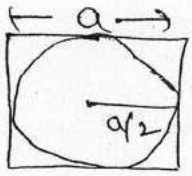
Body Diagonal $= \sqrt{a^2+a^2+a^2} = \sqrt{3}a = D$

2. Total surface area $= 2D^2 = 2(\sqrt{3}a)^2 = 6a^2$

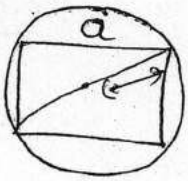
Total surface area $= 6a^2 = 2D^2$

VOLUME $= a^3$

→ Th ... can be kept inside the cube of side 'a' units = $\boxed{\frac{a}{2}}$



→ The radius of the smallest possible sphere in which we can put a cube of side 'a' units = $\boxed{\frac{\sqrt{3}a}{2}}$

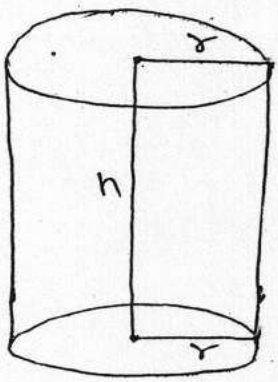


$d = \sqrt{3}a$ = diameter

radius = $\frac{\sqrt{3}a}{2}$

3. CYLINDER

(A) RIGHT CIRCULAR CYLINDER :- (SOLID CYCLINDER)



C.S.A. = Perimeter of base × height (for ALL)

$= 2\pi r \times h$

C.S.A. = $2\pi rh$

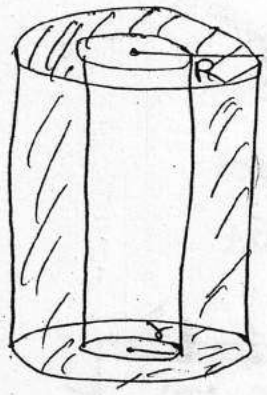
Total surface Area = C.S.A. + 2 × Area of circular region

$= 2\pi rh + 2\times\pi r^2$

$= \boxed{2\pi r(r+h)}$

$= \boxed{\pi r^2 h}$ → Area of base × ht

(b) HOLLOW CYLINDER:-



VOLUME of the SOLID part of hollow cylinder

$= \boxed{\pi (R^2 - r^2) h}$

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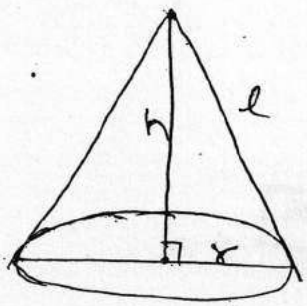
4 CONE:- When a right angle Δ is rotated then a cone is formed.



r = radius of circular base.

l = slant height

h = vertical height

$\boxed{l = \sqrt{h^2 + r^2}}$

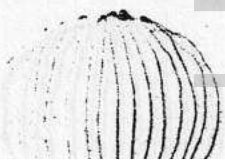
C.S.A. = $\frac{1}{2}$ × Perimeter of base × slant height

$= \frac{1}{2} \times$ circumference $\times l$

$= \frac{1}{2} \times 2\pi r \times l$

$= \boxed{\pi r l}$ $= \boxed{\pi r \sqrt{r^2 + h^2}}$

T.S.A. = $\pi r l + \pi r^2$

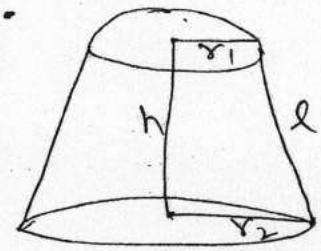


$= \frac{1}{3} \times \pi r^2 \times h$

$= \boxed{\frac{1}{3}\pi r^2 h}$

Volume of cone = $\frac{1}{3}$ x volume of cylinder

FRUSTRUM OF CONE



r_1 = radius of upper circular part

r_2 = radius of lower circular part

h = vertical height

l = slant height

$$l = \sqrt{h^2 + (r_2 - r_1)^2}$$

C.S.A. $= \pi r_1 l + \pi r_2 l = \pi l (r_1 + r_2)$

T.S.A. $= \pi r_1 l + \pi r_2 l + \pi r_1^2 + \pi r_2^2$

$= \pi (r_1^2 + r_2^2 + r_1 l + r_2 l)$

Volume $= \frac{\pi}{3} h (r_1^2 + r_2^2 + r_1 r_2)$

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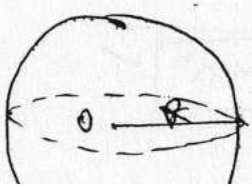
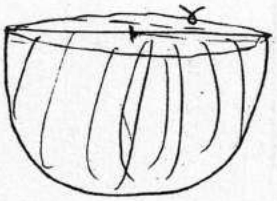
S SPHERE :- A set of all points in space which are equidistant from a fixed point



VOLUME = $\frac{4}{3}\pi R^3$

HEMISPHERE:



C.S.A. = $2\pi r^2$

T.S.A. = $2\pi r^2 + \pi r^2 = 3\pi r^2$

VOLUME = $\frac{2}{3}\pi r^3$

SPHERICAL SHELL:

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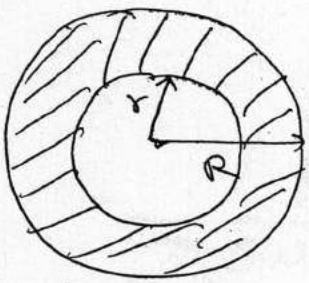


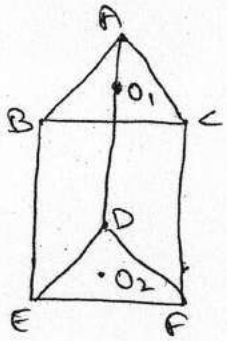
Outer surface Area = $4\pi R^2$

VOLUME OF MATERIAL = $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

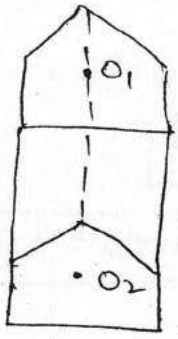
$= \frac{4}{3}\pi(R^3 - r^3)$

regular polygon and faces are rectangular.

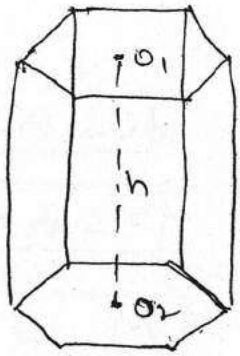
→ Simplest Prism has base of equilateral Δ.



Triangular Prism



Pentagonal Prism



Hexagonal Prism

h = vertical distance b/w midpoints of the bases of a prism

n = no. of sides in regular polygon (base)
= no. of rectangular faces in the prism.

a = each side of base

Slant surface Area (≅ C.S.A.) = Lateral surface Area
= n × Area of each rectangle
= n × (a × h)
= (na) × h

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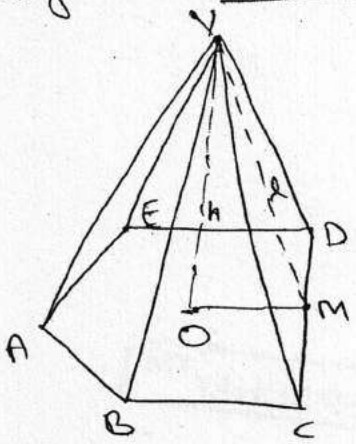
C.S.A. = (Perimeter of base) × vertical height

Total surface Area = Slant surface Area + 2 × Area of base

Volume = Area of base × height

Note:- Cuboid & Cube are also prisms with rectangle and

7 **PYRAMID**:- A pyramid is a solid whose base is a plane rectilinear figure and whose side faces are triangles having a common vertex outside the plane of the base.



Base = Pentagon

so five faces = VAB, VBC, VCD, VDE, VEA

~~Height of a pyramid~~:- length of ⊥r drawn from vertex to the base.

Axis of pyramid:- str. line joining vertex to centre pt. of base.

RIGHT PYRAMID :- ht of pyramid = Axis of pyramid

i.e. if ⊥r dropped from vertex on the base meets the base at. centre pt.

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~~REGULAR PYRAMID~~:- if base is a regular figure. (all sides equal)

[All Δ's are congruent]

Slant height (ℓ) → line segment joining the vertex to the mid pt. of any side of the base.

$$VM = \sqrt{VO^2 + OM^2} = \ell$$

Let n = no. of sides of the base
= no. of triangular faces

h = height of regular pyramid

ℓ = slant height (= ht of each triangular face)

Slant surface area (≈ C.S.A.)

= n × area of one triangle

= $n \times \frac{1}{2} \times$ base $\times \ell$

= $n \times \frac{1}{2} \times a \times \ell$

= $\frac{1}{2}(na) \times \ell$

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C.S.A. = $\frac{1}{2}$ (Perimeter of base) × slant height

Total surface area = Slant surface area + area of base

Volume = $\frac{1}{3}$ × area of base × height

NOTE:- PRISM is equivalent to cylinder

C.S.A.

T.S.A. = C.S.A. + 2 × Area of base

Volume = Area of base × ht.

PRISM

* FOR CONE, PYRAMID

C.S.A. = $\frac{1}{2}$ × Perimeter of base × slant ht.

T.S.A. = C.S.A. + Area of base

Volume = $\frac{1}{3}$ × Area of base × Corr. ht.

Pyramid

* FOR SPHERE – HEMISPHERE

C.S.A. = T.S.A. = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

C.S.A. = $2\pi r^2$

T.S.A. = $3\pi r^2$

Vol. = $\frac{2}{3}\pi r^3$

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* Tetrahedron [combination of 4 equilateral Δ]

[formulae related to Pyramid]

C.S.A. = $\frac{1}{2} \times 3a \times \frac{\sqrt{3}}{2} a$ → slant ht.

$$\text{C.S.A.} = \frac{3\sqrt{3}}{4} a^2$$

T.S.A. = $\frac{3\sqrt{3}}{4} a^2 + \frac{\sqrt{3}}{4} a^2 = \sqrt{3}\, a^2$

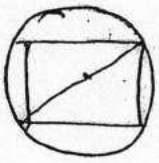
Volume = $\frac{1}{3} \times \frac{\sqrt{3}}{4} a^2 \times \frac{\sqrt{2}}{\sqrt{3}} a = \frac{\sqrt{2}}{12} a^3$

$$\text{ht. of tetrahedron} = \frac{\sqrt{2}}{\sqrt{3}} a$$

① The length, breadth & ht of a cuboid are 12cm, 15 cm & 6 cm resp. Find the longest possible iron-rod that can be kept inside cuboid? (9√5 cm)

$$\left[D = \sqrt{12^2+15^2+6^2} = \sqrt{405} = 9\sqrt{5}\right]$$

② The radius of the smallest possible sphere in which we can put a cube of side 'a' units is 9cm. Find the value of 'a'? (6√3)



$r = 9\text{cm}$, $d = 9\times2 = 18\text{cm}$

Diagonal of cube $= \sqrt{3}a = 18 \rightarrow a = \frac{18}{\sqrt{3}} = 6\sqrt{3}\text{cm}$

③ The length and the breadth of a cuboid are inc. by 20% & 10% resp. The ht. of cuboid is dec. by 15%. Find % change in volume? (+12.2% inc.)

$$20+10-15+\frac{(20)(10)+(20)(-15)+(10)(-15)}{100}+\frac{(20)(10)(-15)}{10,000}$$

$$15-\frac{250}{100}-0.3 = +12.2\%\text{ inc.}$$

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④ A brick measures 20cm × 10cm × $\frac{15}{2}$ cm. How many such bricks will be required for a wall 25m × 2m × $\frac{3}{4}$ m? (25000)

Vol. of wall $= 25\times2\times\frac{3}{4}\ m^3$, Vol. of 1 brick $= \frac{20}{100}\times\frac{10}{100}\times\frac{15}{200}\ m^3 = \frac{3}{200}\ m^3$

No. of bricks $= 25\times2\times\frac{3}{4}\times\frac{200}{3} = 25000$

⑤ Find the volume of a cuboid whose areas of base and two adjacent faces are 180 cm², 96 cm² & 120 cm² resp? (1440 cm³)

$$V = \sqrt{A_1 \cdot A_2 \cdot A_3} = \sqrt{180\times96\times120} = 1440\text{ cm}^3$$

if thickness ... m. ... capacity of box? (240 cm³)

$l = 9 - 2\times0.5 = 8$ cm, $b = 7 - 2\times0.5 = 6$ cm, $h = 6 - 2\times0.5 = 5$ cm.

Capacity $= 8\times6\times5 = 240\ cm^3$

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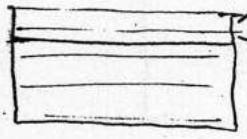
(7) A rectangular tank is 50 m long & 29 m deep. if 1000 cubic metres of water be drawn off the tank, the level of water goes down by 2 metres. How many cubic mtrs of water can the tank hold? (14500 m^3)



$V = l\times b\times h$

$1000\ m^3 = 50\times2\times b \rightarrow b = 10$ m.

$V = 50\times10\times29 = 14500\ m^3$

---

(8) Three cubes of metals of edges 3 cm, 4 cm & 5 cm resp. are melted & formed into a single cube. Find the side of new cube? (6cm)

$V_1 + V_2 + V_3 = V \rightarrow 27 + 64 + 125 = 216$

side of new cube $= \sqrt[3]{216} = 6$ cm

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(9) A cube of side 3cm is melted and smaller cubes of sides 1cm each are formed. How many such cubes are possible? (27)

$V = n\times V_1 \rightarrow V = 27\ cm^3 \rightarrow 27 = n\times1$

$n = 27$

---

(10) A stream which flows at a uniform rate of 2.5 km/hr, is 20 mtrs wide and the depth of a certain ferry being 1.2 m. How many ltrs pass the ferry in a minute? (1000000 ltrs)

$V = l\times b\times h$

$l = 2.5$ km/hr $= \frac{125}{3}$ m/min.

$b = 20$ m, $h = 1.2$ m

(11) ... surface area

$-50-50+\frac{2500}{100}=-75\%$ = 75% dec. in surface area

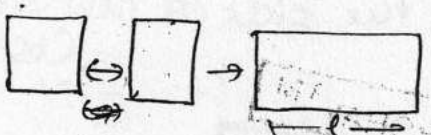
$-75-50+\frac{75\times50}{100}=-87.5\%$ = 87.5% dec. in volume

(12) The volumes of two cubes are in the ratio of 8:125. Find the ratio of their edges and surface areas?

$a_1^3 : a_2^3 = 8:125 \rightarrow a_1 : a_2 = \sqrt[3]{8} : \sqrt[3]{125} = 2:5$ (edges)

$a_1^2 : a_2^2 = 2^2 : 5^2 = 4:25$ (surface areas)

(13) Two cubes each of edge 10m are joined to form a single cuboid. What is surface area of new cuboid formed? (1000 cm^2)



$l = 2\times10 = 20m$

$b = 10m$, $h = 10m$

surface area $= 2(lb+bh+lh) = 2(200+100+100)$

$= 1000 cm^2$

(14) Three cubes of edges 3cm, 4cm and 5cm resp. form a single cube. Find surface area of new cube? (216)

$V = 3^3+4^3+5^3 = 216 \rightarrow a = \sqrt[3]{216} = 6$

surface area $= 6a^2 = 6\times6\times6 = 216\ cm^2$

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(15) The diagonal of a cube is $15\sqrt{3}$ cm. Find the ratio of its total surface area and volume? (2:5)

$D = \sqrt{3}a = 15\sqrt{3} \rightarrow a = 15$ cm

$6a^2 : a^3 \rightarrow \frac{15\times15\times6}{15\times15\times15} = \frac{2}{5}$

① The C.S.A. of a cone is 4070 cm² and its circumference is 70 cm. Find slant height of cone? (37 cm)

$\left[\pi r l = 4070 \rightarrow l = 4070 \times \frac{7}{22} \times \frac{1}{35} = 37 \text{ cm}\right]$

② There are two cones. The C.S.A. of one is twice that of the other. The slant height of the latter is twice that of the former. Find the ratio of their radii? (4:1)

$\left[\pi r_1 l_1 = 2\pi r_2 \times 2l_1 \rightarrow r_1 l_1 = 4 r_2 l_1 \rightarrow r_1 = 4r_2 \rightarrow r_1 : r_2 = 4:1\right]$

③ A hollow cylindrical tube open at both ends is made of iron 2cm thick. If the external dia. be 50cm and length of tube be 140 cm, find the volume of iron in it? (42240)

Volume of iron $= \frac{22}{7} \times 140 \times (25^2 - 23^2)$

$= 42240 \text{ cm}^3$

④ Find what length of canvas, 2 mtrs in width, is required to make a conical tent, 8 mtrs in dia. and 5.6 mtrs in slant height? (35.2 mtr)

$\left[\pi r l = \frac{22}{7} \times 4 \times 5.6 = 70.4 \text{m}^2 = L \times b\right]$

$L = \frac{70.4}{2} = 35.2 \text{ mtr.}$

⑤ The diameter of a right circular cone is 10 mtrs and its slant height is 13 mtrs. Find its 1) C.S.A. 2) T.S.A. 3) volume?

C.S.A. $= \pi r l = \frac{22}{7} \times \frac{10}{2} \times 13 = 204.3 \text{ m}^2$

T.S.A. $= \pi r(r+l) = 282.6 \text{m}^2$

Volume $= \frac{1}{3}\pi r^2 h \rightarrow h = \sqrt{169 - 25} = 12$

of 1: ... ratio ... (√2 : 1)

$\pi r_1^2 h_1 = \pi r_2^2 h_2 \rightarrow r_1^2 h_1 = r_2^2 2h_1 \rightarrow \left(\frac{r_1}{r_2}\right)^2 = 2$

$r_1 : r_2 = \sqrt{2} : 1$

(7) if the heights of two cones are in the ratio 1:4 and their diameters are in the ratio 4:5, what is the ratio of volumes? (4:25)

$\frac{1}{3}\pi(4r_1)^2 h_1 : \frac{1}{3}\pi(5r_1)^2 \times 4h_1$

$16 r_1^2 h_1 : 25 r_1^2 \times 4h_1$

$4 : 25$

(8) if the radius of a cylinder is doubled and the height is halved, what is the ratio b/w new volume & previous volume? (2:1)

~~New : Previous~~ → Previous : New →

$\pi r_1^2 h_1 = \pi (2r_1)^2 \cdot \frac{h_1}{2}$

$\Rightarrow r_1^2 h_1 = 4 r_1^2 \cdot \frac{h_1}{2}$

$\Rightarrow 1 : 2$

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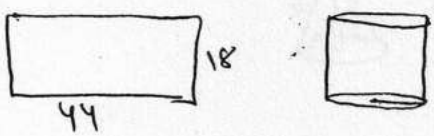
(9) Radius and height of a right circular cylinder is both inc. by 10%. Find % change in volume? (33.1% inc.)

$10 + 10 + \frac{100}{100} \rightarrow 21$

$21 + 10 + \frac{210}{100} \rightarrow 33.1\%$ inc.

(10) A rectangular sheet of 44cm × 18cm is rolled along its length and a cylinder is formed. Find the volume of cylinder?



$2\pi r = 44$

$r = 44 \times \frac{7}{22} \times \frac{1}{2} = 7\text{cm}$

Volume $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 18 = 2772\ \text{cm}^3$

are ... same radius of base. find the ht. of the cone? (9)

[vol. of cone = vol. of cylinder
$\frac{1}{3}\pi r^2 h = \pi r^2 \times 3 \rightarrow h = 9$ cm]

(10) The heights of a cone, cylinder and hemisphere are equal and their radii are in ratio 2:3:1, then find ratio of their volumes?

[$\frac{1}{3}\pi r_1^2 h : \pi r_2^2 h : \frac{2}{3}\pi r^3$
$\frac{(2r)^2}{3} : (3r)^2 : \frac{2}{3}r^2$
$\frac{4r^2}{3} : 9r^2 : \frac{2}{3}r^2$
$4 : 27 : 2$]

[ht. of cone = radius of hemisphere
or ht. = ht of cylinder]

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(13) A solid cylinder has total surface area of 462 cm^2 and its C.S.A. is $\frac{1}{3}$rd of its T.S.A. find volume of cylinder? (539 cm^3)

[$2\pi r^2 + 2\pi rh = 462$, $2\pi rh = \frac{1}{3} \times 462 = 154$
$2\pi r^2 + 154 = 462$
$r^2 = 49 \rightarrow r = 7$cm
$2\pi rh = 154 \rightarrow h = \frac{7}{2}$ cm.
volume $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times \frac{7}{2} = 539$ cm^3]

(14) A cylinder and a cone have equal radii of their bases and equal hts. if their C.S.A. are in the ratio 8:5. find the ratio of their radius and ht? (3:4)

[$\frac{2\pi rh}{\pi r\sqrt{h^2+r^2}} = \frac{8}{5} \rightarrow \frac{h}{\sqrt{h^2+r^2}} = \frac{4}{5} \rightarrow \frac{h^2}{h^2+r^2} = \frac{16}{25} \rightarrow \frac{h^2+r^2}{h^2} = \frac{25}{16}$
$1 + \frac{r^2}{h^2} = \frac{25}{16} \rightarrow \frac{r^2}{h^2} = \frac{9}{16} \rightarrow \boxed{\frac{r}{h} = \frac{3}{4}}$]

① Find ... 21 cm?

[Volume = 19404 cm³, C.S.A. = 2772 cm², T.S.A. = 4158 cm²]

② Find the no. of lead balls of diameter 1 cm each that can be made from a sphere of dia. 16 cm? (4096)

$$\left[\frac{\frac{4}{3}\times\pi\times 8\times 8\times 8}{\frac{4}{3}\times\pi\times 0.5\times 0.5\times 0.5} = 4096\right]$$

③ How many spherical bullets can be made out of a lead cylinder, 28 cm high and 6 cm radius and each bullet being 1.5 cm in dia.? (1792)

$$\left[\text{no.} = \frac{\text{vol. of cylinder}}{\text{vol. of one bullet}} = \frac{\pi\times 6\times 6\times 28}{\frac{4}{3}\times\pi\times(0.75)^3} = 1792\right]$$

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④ The radii of two spheres are in the ratio of 1:2. What is the ratio of their surface areas? (1:4)

$$\left[4\pi r_1^2 : 4\pi(2r_1)^2 \rightarrow r^2 : 4r_1^2 \rightarrow 1:4\right]$$

⑤ The C.S.A. of two spheres are in the ratio 1:4. Find the ratio of their volumes? (1:8)

$$\left[\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{1}{4} \rightarrow \frac{r_1}{r_2} = \frac{1}{2} \Rightarrow \frac{4}{3}\pi r_1^3 : \frac{4}{3}\pi(2r_1)^3 \rightarrow r^3 : 8r_1^3 \rightarrow V_1 : V_2 \rightarrow 1:8\right]$$

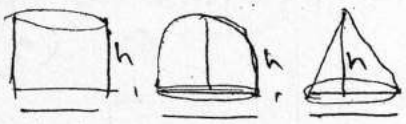
⑥ The radius of a sphere is inc. by 5%. Find % inc. in its surface area & volume?

[Surface area → $4\pi r\times r$ → $5+5+\frac{25}{100}$ = 10.25% inc.

Vol. → $\frac{4}{3}\times\pi\times r\times r\times r$ → 10.25 + 5 + $\frac{10.25\times 5}{100}$ = 15.7625% inc.]

Sam

Let x cm → dia of base, h cm → ht.



~~cylinder → radius = $\frac{x}{2}$, ht = ~~

hemisphere → radius = $\frac{x}{2}$, ht, $h = \frac{x}{2}$ (radius = ht.)

cone → $r = \frac{x}{2}$, $h = \frac{x}{2}$

cylinder → $r = \frac{x}{2}$, $h = \frac{x}{2}$

vol. of cylinder : volume of hemisphere : volume of cone

$\pi\left(\frac{x}{2}\right)^2\left(\frac{x}{2}\right) : \frac{2}{3}\pi\left(\frac{x}{2}\right)^3 : \frac{1}{3}\pi\left(\frac{x}{2}\right)^2\left(\frac{x}{2}\right)$

$1 : \frac{2}{3} : \frac{1}{3}$ → **3 : 2 : 1**

or $\pi x^2 h : \frac{2}{3}\pi x^3 : \frac{1}{3}\pi x^2 h$

(8) if radius of a sphere is inc. by 20%, find % inc. in volume?

$20 + 20 + 20 + \frac{4000 + 4000 + 4000}{100} + \frac{8000}{10000} = 60 + 12 + 0.8 = 72.8\%$

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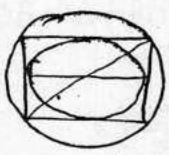
(9) The dia. of a sphere is $500\sqrt{3}$ cm. A cube of largest size is kept inside it and again a sphere of largest size is kept inside cube. Find the ratio of volume of larger cube to that of small sphere? (6 : π)



Diagonal of cube = $500\sqrt{3} = \sqrt{3}a$

a = side of cube = 500 cm.

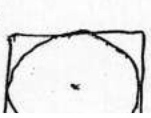
Dia. of small sphere = a = 500cm, radius = 250 cm.

volume of cube : vol. of small sphere

$(500)^3 : \frac{4}{3}\pi(250)^3 \rightarrow 125000000 : \frac{4}{3}\pi \times 15625000$

$\frac{125000 \times 3}{4 \times 15625 \times \pi} = \frac{6}{\pi}$

(10) Find the ratio of the volume of a cube to that of a sphere, which will exactly fit inside the cube? (6 : π)



$a^3 : \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$

$a^3 : \frac{4}{3}\pi\frac{a^3}{8}$

PRISM

(1) Base of a right pyramid is a square and length of diagonal of the base is $24\sqrt{2}$ m. if the volume of the pyramid is 1728 m^3. Find its height? (9 mtrs)

[Area of base $= \frac{1}{2} \times d^2 = 576\ m^2$ or $a^2 = 24 \times 24 = 576\ m^2$

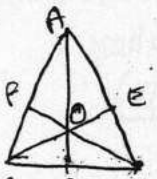
volume $= \frac{1}{3} \times$ ht $\times$ area of base $\rightarrow 1728 = \frac{1}{3} \times h \times 576 \rightarrow h = 9$ mtrs.]

(2) Base of a rt. prism is an equilateral Δ of side 6 cm. if the volume of prism is $108\sqrt{3}$ cm^3, find its height? (12 cm)

[Area of base $= \frac{\sqrt{3}}{4} \times 6^2 = 9\sqrt{3}\ cm^2$

volume = Area of base $\times h \rightarrow 108\sqrt{3} = 9\sqrt{3} \times h \rightarrow h = 12$ cm]

(3) The perimeter of the triangular base of a rt. prism is 15 cm and radius of incircle of triangular base is 3 cm. if volume of prism is 270 cm^3, find the ht. of prism? (12 cm)



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$OE = OD = OF = 3$ cm

Area of base $= \frac{1}{2} \times AB \times OF + \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OE = \frac{1}{2} \times 3 \times (AB + BC + AC)$

$= \frac{1}{2} \times 3 \times 15 = \frac{45}{2}\ cm^2$

vol. $\Rightarrow 270 = \frac{45}{2} \times h \rightarrow h = \frac{270 \times 2}{45} = 12$ cm

(4) The base of a right pyramid is a square of side 40 cm long. if volume of pyramid is 8000 cm^3, find its height? (15 cm)

[vol. $= \frac{1}{3} \times$ base area $\times h \rightarrow \frac{1}{3} \times 1600 \times h = 8000 \rightarrow h = \frac{8000 \times 3}{1600} = 15$ cm]

(5) 8 cm. ... if volume of prism is 1056 cm^3, find ht. of prism? (12 cm)

Vol. = Base area × ht → $\frac{1}{2}(14+8) \times 8 \times h = 1056$

$h = \frac{1056}{88} = 12$ cm

(6) The base of a right pyramid is a square of side 16 cm long. if its ht be 15 cm then find the area of the lateral surface? (544 cm^2)

Lateral surface area = $\frac{1}{2}$ × Perimeter of base × slant ht.

slant ht. = $\sqrt{15^2+8^2} = 17$ cm.

L.S.A. = $\frac{1}{2} \times 64 \times 17 = 544$ cm^2

(7) The base of a right prism is an equilateral Δ of side 8 cm and ht. of prism is 10 cm. Find volume of prism? ($160\sqrt{3}$ cm^3)

Volume = Area of base × ht

$= \frac{\sqrt{3}}{4} \times 64 \times 10 = 160\sqrt{3}$ cm^3

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(8) The base of a right prism is an equilateral Δ of area 173 cm^2 and volume of prism is 10380 cm^3. Find the area of lateral surface of prism? ($\sqrt{3} = 1.73$) (3600 cm^2)

Vol. = 173 × h → $h = \frac{10380}{173} = 60$ cm

Area of Δ = $\frac{\sqrt{3}}{4}a^2$ → $173 = \frac{\sqrt{3}}{4}a^2$ → $a = 20$ cm.

L.S.A. = Perimeter of base × ht = 60 × 60 = 3600 cm^2

	C.S.A.	T.S.A.	VOLUME
CUBOID	$2h(l+b)$	$2(lb+bh+lh)$	$l \times b \times h$
CUBE	$4a$	$6a$	a^3
SPHERE	$4\pi R^2$	$4\pi R^2$	$\frac{4}{3}\pi R^3$
HEMISPHERE	$2\pi R^2$	$3\pi R^2$	$\frac{2}{3}\pi R^3$
CYLINDER	$2\pi Rh$	$2\pi r(r+h)$	$\pi r^2 h$
CONE	$\pi r l$	$\pi r(r+l)$	$\frac{1}{3}\pi r^2 h$
FRUSTRUM OF CONE	$\pi r_1 l + \pi r_2 l$	$\pi r_1 l + \pi r_2 l + \pi r_1^2 + \pi r_2^2$	$\frac{\pi}{3}h(r_1^2+r_2^2+r_1 r_2)$
PRISM	Perimeter of base × height	C.S.A. + 2 × Area of base	Area of base × height
Pyramid	$\frac{1}{2}$ × Perimeter of base × slant height	C.S.A. + Area of base	$\frac{1}{3}$ × Area of base × height.
Tetrahedron	$\frac{3\sqrt{3}}{4}a^2$	$\sqrt{3}\,a^2$	$\frac{\sqrt{2}}{12}a^3$

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